

**Progressivity, Equity and the Take-up of State
Benefits, with Application to the 1985 British Tax
and Benefit System**

by

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Ph.D. Thesis (Economics)

October 1992

**London School of Economics and Political Science
University of London**

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Abstract

We investigate the allocation and the effects of personal taxes and state and social security benefits in modern economies. The most prominent contributions are as follows:

(1) We propose a unified framework in which to discuss the progressivity, redistribution and equity of taxes and benefits. Through this, we offer a general class of indices of horizontal inequity that complements existing classes of progressivity and vertical equity indices. We highlight throughout the analytical and empirical contribution of individual taxes and benefits to the effect of the whole system, using the features of the 1985 British tax and benefit system.

(2) We analyse state benefit take-up and welfare programme participation in the presence of divergences between the assessment of entitlement made by the take-up analyst and that carried out by the government's agency. This explicit modelling helps remove important biases in the computation of take-up and participation rates. It also detects the presence of allocative errors made by the government in alleviating poverty. Our methodology -- which may be usefully extended to other microeconomic applications -- simultaneously identifies the distribution of costs to participating in welfare programmes.

(3) We provide econometric evidence on the level of claiming inconveniences inherent to the British Supplementary Benefit (now Income Support) programme and on how they dampen the welfare impact of state support. Besides, we can illustrate the degree of misallocation of state support among the poor and the non-poor. These allocative errors are also respectively aggravated and mitigated by the deterrence effect of claiming costs. We also

examine the impact of allocative imperfections upon the level of progressivity, equity and redistribution exerted by redistributive tools.

(4) We model the optimal design of state support in the presence of heterogeneity in original incomes and in the costs incurred in granting state support. We see that some simple rules which hold when income redistribution and poverty alleviation are costless do not hold anymore in more general cases. This also has important consequences for the consideration of principles of vertical and horizontal equity.

**A Hélène, mon adorable fiancée,
et à mes parents, qui m'ont tout donné**

Acknowledgements

Many thanks must go to Tony Atkinson, my benevolent doctoral supervisor, and to Dave Coady, Jane Dickson, Stephen Edward, Joanna Gomulka, John Hills, Stephen Howes, Peter Lanjouw, Holly Sutherland and Brian Warren who kindly and variously offered their advice and support throughout the too short years of my Ph.D.. I am grateful to the Central Statistical Office, the Office of Population Censuses and Surveys and the ESRC Data Archive for permission to use FES data. I have also benefitted from the financial support of the Commonwealth Scholarships Commission and from the logistical one of STICERD.

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Introduction

This thesis analyses the allocation and the effects of personal taxes and state and social security benefits in modern economies. Our aim is twofold.

Firstly, we focus our attention on the depiction of the progressivity, redistribution and equity of taxes and benefits. Our approach to such issues is primarily methodological and we indicate particularly how progressivity and vertical and horizontal equity may or may not yield insights into the desirability of the government's redistributive actions. We also illustrate the application of our methodological tools using a personal tax and benefit model similar to the many now widely found in government departments and research institutions. Through this, we can replicate and enhance the ways in which tax and benefit systems and reforms are often assessed.

Secondly, we extend this traditional analysis to incorporate the presence of imperfect information and allocative costs into the empirical, methodological and theoretical analysis of the state's redistributive tools. Imperfect information prevents the state from targeting precisely its taxes and benefits among the diverse members of a society; this leads to inequities and inefficiencies in the use of the state's redistributive tools. Imperfect information also hinders a tax and benefit analyst from monitoring accurately the allocation and the impact of taxes and benefits in a population; when unchecked, this feature can generate significant biases in the normative and positive conclusions reached. Allocative or redistributive costs, which can also limit the effectiveness of government policy, easily arise, for instance, from the existence of administrative, claiming (for benefits) or enforcement (for taxes) costs. These imperfections have, among other

things, important consequences for the optimal design of redistributive policy. We detect the empirical presence of such redistributive costs and imperfect information by deriving methodological tools that account for their existence. We also note their theoretical and empirical impact on progressivity, redistribution and equity as discussed in the context of the more conventional tax and benefit analysis.

All of the applications are on the tax and benefit system prevailing in Britain in 1985, although any other tax and benefit system combined with sufficiently detailed data could equally well have been chosen. The analytical tools developed and subsequently applied are thus universal, and so is probably the interpretation of some of the empirical results shown for Britain throughout the following chapters.

* * *

Chapter I introduces a computer model that constitutes an essential building stone to the empirical work offered by the thesis. It is there that we describe relatively briefly the data used, the approach taken to translate tax and benefit rules into a set of earning opportunities for a sample of households, and the features of the 1985 British tax and benefit system that were embedded in the model. In the second part of Chapter I, we shed light on the shape, irregularities and nonlinearity of the derived budget sets of households, we identify the distribution of marginal implicit tax rates across the population and we examine how good an approximation is provided by linearisations of the budget constraint at the point of actual labour supply. Apart from bringing forth some useful methodological contributions, this exercise also helps to illustrate the complex web

of taxes and benefit regulations that underlie implicitly the basis of the empirical results of the following chapters.

Chapter II displays a framework in which the progressivity, redistribution and equity of taxes and benefits can be systematically discussed and measured, both separately and in relationship to the other two concepts. The first section concentrates on measures of progression at specific points of budget constraints, and on how we may distinguish the separate effects of taxes and benefits. Using as a measuring rod a standard social welfare function, we attempt to quantify the social value of progression and we see how we may approximate the intricate 1985 system to one with a constant degree of tax and benefit progression. The second section of the chapter extends the analysis to global progressivity and redistribution, emphasising again the separate contribution of various taxes and benefits. We concentrate our attention on a class of measures for which we subsequently see how total redistribution can be decomposed into a function of the degree of progressivity ("vertical equity") and of reranking ("horizontal inequity") achieved by each of various taxes and benefits. Having derived indices of redistribution and horizontal inequity, we may also combine them to yield indicators of the desirability of existing and simulated tax and benefit systems. The analytical tools developed are once again applied to the British tax and benefit system of 1985.

The material of Chapter III first discusses extensively some of the concepts and the possible structure of an analysis of the take-up of state benefits. We are then able to derive an implementable econometric model that can help enhance our understanding of the take-up of state benefits in many significant ways. We

first ask whether the use of imperfect data to compute or predict eligibility may explain the observation of less than full benefit take-up rates. With this in mind, we then see how to justify and conduct the analysis of benefit take-up in the presence of divergences and errors in modelling entitlement. Among other things, this explicit modelling goes a long way towards removing important biases in the computation of take-up rates. Our methodology, which may be extended to other microeconomic applications, allows for the relative and absolute parametric identification of the distributions of observable and unobservable costs to claiming and of those of entitlement discrepancies. In particular, the procedure can readily yield direct estimates of the monetary equivalents to the burden of seeking state support.

Chapter IV applies the analysis of Chapter III to some Family Expenditure Survey data gathered on the take-up of Supplementary Benefits in 1985 Britain. We discuss the survey data and we allow for the inclusion of the pensioners and the self-employed -- for whom survey deficiencies have been repeatedly documented. We then present the results of our estimation over the sample, examining in particular the estimated "entitlement discrepancies" and "costs to claiming". We look at the validity of various empirical definitions of take-up statistics, considering among other things how the estimation of the level of entitlement as assessed by the government agency can amend our understanding of such figures.

The last chapter provides both a theoretical and an applied discussion of the impact of imperfections in the administration of state benefits upon the optimal allocation of benefits and on social welfare and equity. We derive the

optimal design of state support in a world where there exist known administrative or claiming costs to redistributing income. We provide empirical evidence on the efficiency and efficacy of government support in providing net benefits and thus in influencing the level of "social welfare". We subsequently look at the desirability of some marginal and major changes in government redistributive policies, discussing briefly a few other considerations involved in an optimal redistributive design. Finally, we use the material presented in Chapter II to develop both a general and an empirical analysis of the impact of allocative imperfections upon the level of progressivity, vertical equity, horizontal inequity and redistribution exerted by redistributive tools.

Each chapter is terminated by a conclusion which outlines very briefly its main findings. The final conclusion brings forward the chief contributions of the thesis and discusses their limits. As an epilogue to the whole work, it also indicates avenues for some useful further ventures.

Chapter I: A Computer Model of the 1985 British

Personal Tax and Transfer System

Introduction

The analysis of personal tax and benefit systems has mostly and traditionally been concerned with the issues of vertical equity (redistributive justice), horizontal equity ("equal treatment of equals" or "maintenance of relative positions"), their effects on various incentives (to work, to save, to invest in one's human capital development, etc.), and on economic efficiency (e.g., the extent to which inefficiency costs are introduced by tax and benefit rules).

We present in section A a computer model adapted to the 1985 British personal taxation and transfer system that may be seen as an important building block towards the discussion of many of these issues in this thesis¹. All tax and transfer modules are independently constructed and only need to be added to yield the final budget set (or subsets of it). Because of this, the model is very flexible and can handle all types of modifications and simulations, starting from the simple addition of a lump-sum tax or benefit to complicated forms of transfers which may be dependent on income unit characteristics, may be taxable or not, may be assessed against gross income or income net of various taxes and benefits, etc.. The computer programme is also capable of incorporating all types of discontinuities in the various modules leading to the net income function, and

¹ For a description of three other tax and benefit models, the TAXMOD, the Institute for Fiscal Studies and the SYSIFF French tax-benefit model, consider Atkinson and Sutherland (1988). A tax and benefit model for the Québec economy can be found in Fortin et al. (1989), and the application of one for Sweden, in Schwarz and Gustafsson (1991).

also allows as many nonconvexities in the budget set and subsets as are required. A novel element is that of extending efficiently the arithmetical analysis of previous models into the complete space of *possible* labour supply and net income. In section B, we will present an illustrative view of the way in which our computer model throws light on the shape and irregularities of the resulting budget sets, on the distribution of explicit and implicit marginal tax rates, and on how good an approximation is provided by linearisations of the budget constraint at the point of *actual* labour supply. Apart from spurring some theoretical considerations on optimal tax and benefit design, this chapter also provides important insights on some of the features of the British tax and benefit system, features that underlie the basis of our empirical work in this thesis.

A- The Model

1- The Data

We use all of the 4471 income units figuring in the Family Expenditure Survey (FES) data from April 8th to October 7th, 1985, and which gather a total of 9043 individuals, of whom 4548 are adults (mostly spouses) living together. The "Statistical Notes on the Use of the 1985 Family Expenditure Survey Data" at the end of the thesis describe the derivation and the use of these data in our study.

We will sometimes have recourse to "typical" income units to illustrate the development of our analysis. Such procedures warrant a great deal of caution. The use of averages and sample groups may give the misleading impression of substantial homogeneity in the population: much of our analysis is in fact to show the extent to which differences in social and economic characteristics influence the role of the tax and transfer systems. Notwithstanding these remarks, we have

defined four income unit types reflecting the average value of the characteristics of their corresponding group. In Table 1.1 we thus indicate some of the average characteristics of a

- (1) family of four, owner-occupiers;
- (2) family of four, renters;
- (3) pensioner couple, owner-occupiers;
- (4) family of four, of which the head is unemployed, owner-occupiers.

A family of four is taken to denote a couple and two dependent children, where these are as defined in Appendix A. A pensioner couple is one in which either the husband is 65 or above, or the wife is 60 or more. The head of a unit is, under the FES definition, the man in the presence of a couple. We note in Table 1.1 that the personal allowance of the pensioner couple is greater because of the extra age allowance. The amount of composite tax retained depends on the level of savings of the respective units. Eligible rates and rents under the Housing Benefit regime vary widely with the nature of the units, and Supplementary Benefit requirements in (1) exceed those in (2) because of the consideration of mortgage interest payments for owner-occupiers such as (1). Hourly wages are as defined in Appendix A, and their missing values have been estimated for those not in paid work. Appendix A also indicates the way in which FES variables have been used to compute some other values of Table 1.1 and, in general, the parameters of the tax and benefit model.

2- The Analysis

We denote the exogenous gross wage by w , weekly hours of work by h , and exogenous gross non-labour income by y . Thus, gross labour income is wh ,

and gross income, denoted by the function $X(wh)$, is $X(wh) = wh + y$.

For each suitable adult, we construct from m tax and benefit provisions m ($i=1,m$) matrices $[(h_{j,i}, t_{j,i}, YV_{j,i})]$, $j=1,J_i$ of size $(J_i, 3)$, with $h_{j-1,i}$ and $h_{j,i}$ being respectively the lower and upper boundaries for the applicability of the vector $(h_{j,i}, t_{j,i}, YV_{j,i})$. Income net of tax or benefit i is N_i . For the interval $[h_{j-1,i}, h_{j,i}]$, $t_{j,i}$ is the applicable marginal tax rate, $w(1-t_{j,i})$, the virtual wage [the slope of the budget constraint in the (hours of work, net income) space], and $YV_{j,i}$, the virtual income [the intercept of the budget constraint in the (h,N) space]. Each "notch" (or discrete jump or fall in the level of net income due to a discrete change in the level of benefit entitlement or tax payable) generates an additional interval with separate vector $(h_{j,i}, t_{j,i}, YV_{j,i})$, where, because of the discontinuity in the net income function, $YV_{j,i} + h_{j-1,i}wt_{j,i} \neq YV_{j-1,i} + h_{j-1,i}wt_{j-1,i}$. Together these $(\sum J_i)$ vectors will jointly determine the shape of the feasible budget set for each suitable adult.

Taking one vector $(h_{j,i}, t_{j,i}, YV_{j,i})$ and leaving out the subscripts, we may define the corresponding net income as

$$N(wh) = y + y^* + (1-t) wh \quad (1)$$

This is depicted on Figure 1.6b, where the horizontal axis shows hours of work, and the vertical one plots income per week. The filled line indicates the net income function, which differs from gross income by the amount of net taxes payable or benefits due. Net taxes are then equal to

$$T(wh) = -y^* + t wh \quad (2)$$

Virtual income YV amounts to $y^* + y$, and defines the vertical intercept of a budgetary linear segment in the $[h, N(wh)]$ space. This is also illustrated in Figure 1.6b. As we shall see in an example below, y^* reflects tax on non-labour income y and provides an index of departure of the tax system from pure proportionality of income tax to income. Reranking across the income space wh of individuals due to the application of the tax could then occur if and only if the derivative of T with respect to wh exceeded one, or if downward notches (discrete falls in the level of net income as wh rises) occurred.

As is customary, we define a progressive tax (benefit) system as one in which the average rate of tax (benefit) increases (decreases) with gross income [see Jakobsson (1976)]. Since the average tax rate, $a(N)$, equals

$$a(N) = T(N)/X = \frac{-y^* + t wh}{X(wh)} = t - \frac{y^* + ty}{X(wh)} \quad (3)$$

we note that, given t , $a(N)$ will increase with gross income $X(wh)$ only if there is a positive "net progressivity effect" in the second term of the previous equation, that is, if

$$y^* > -t y \quad (4)$$

It is equivalent to requiring that the effect on net income of a departure from pure proportionality be superior (more beneficial) than that of simply taxing all non-labour income at a constant rate t .

We cannot help but refer readers to Hausman (1985) for an interesting and fuller discussion of the way in which the vectors (h_j, t_j, YV_j) may be built and of

how they interact together. We provide one simple instance. If agents only face a segmentary linear income tax (thus, $m=1$) such that explicit marginal tax rates t_j are applied to the brackets of taxable incomes $[YL_{j-1}, YL_j]$, with $YL_0=0$ and with corresponding hours-of-work intervals $[h_{j-1}, h_j]$, such that $YL_j = h_j \cdot w + y_T \forall j$ ², then net income N_j with h falling within $[h_{j-1}, h_j]$ equals

$$N_j = y + w \cdot h - \sum_{k=1}^j (t_k - t_{k-1}) (y + w \cdot h - YL_{k-1}) = w \cdot h \cdot (1 - t_j) + YV_j \quad (5)$$

with $t_0=0$. $w(1-t_j)$ is the slope of the budget constraint in the (h, N) space, and YV_j is its intercept and depends on the size of y and y_T , and on the characteristics of the segmentary income tax system:

$$YV_j = y + y^* = y + \left[-t_j y_T + \sum_{k=1}^j (t_k - t_{k-1}) (YL_{k-1}) \right] \quad (6)$$

In practice, of course, the complexity of the income tax system is increased by, for instance, various additional tax deductions, surcharges or credits. The arguments of a more general tax function would also include a vector of income unit characteristics other than income. The analysis made below includes all of the significant components of the personal income tax system, and captures fully the effects of the major social security and state benefits in 1985 Britain. These components and those of other years are well described in the Tolley's Income Tax and Social Security and State Benefits yearly guides. Some benefits are not allocated by the model but are granted to the income unit when declared in the

² y_T is taxable non-labour income.

survey. They include widow's benefits, various contributory and non-contributory injury and disablement pensions and allowances, and transfers under the training opportunities scheme: together they represent a proportion of 4.8% of gross family income³. The programme does not, however, encompass any of the benefits made "in kind" by the state, such as those provided by health and education expenditures or by "passport benefits" arising from the grant of cash benefits (e.g., that of supplementary benefits), nor does it incorporate general housing subsidies (mainly to council tenants), local authority taxes or indirect taxes.

The analysis takes advantage of the recursivity of the tax and benefit system in Britain (which also makes the administration of the programmes much simpler) and is illustrated in Table 1.2 and described in more details below. First, having computed a level of original or gross income and observed the receipt of various benefits (described in Appendix A), we allocate child and one-parent benefits (CB, OPB). Once this is done, we consider the module of those "means-tested" benefits for which entitlement depends solely on gross incomes and hours worked. These transfers include the unemployment (UNB) and the retirement pension (BP) contributory benefits. We then construct the independent effects of the taxation of personal incomes (which include the previous taxable contributory benefits), and eventually move on to the provision of supplementary benefits, which depend recursively on income *net of income taxes*, etc..

³ Unless specified, figures relate to calculations made on our FES sample. For many of the benefits modelled, the grossed-up number of recipients and levels of expenditures seem to agree very much with the published administrative aggregates. We discuss this further later in the chapter.

3- The Building Blocks

The taxes and transfers modelled are as follows. All statements and figures refer to the 1985 sampling period defined above. The major changes to the British tax and benefit system between 1985 and 1992 are summarised in Appendix B.

(a) Child Benefit (CB) and One-Parent Benefit (OPB)

These are not means-tested, and amount to 2.7% of gross family income.

(b) Unemployment Benefits (UNB)

These are available to those income units whose adults are not susceptible of receiving (c) and include, when appropriate, additional benefits for dependants. UNB are also subject to "earnings rules" for own eligibility and for the eligibility to the increase for dependants. For some of our subsequent analysis in this and the following chapter, we would like to illustrate the changes in net income and in the progressivity of the tax and benefit system when hours of work and gross income vary. Because of this, we need assumptions on the grant of UNB to those currently working but who might end up unemployed once we vary their labour market behaviour. Similar assumptions are required for those unemployed who may (according to our simulations) enter the labour market. As we dispose of no FES contributory records, we will make the working assumptions that no UNB may be paid if the person is not *currently* receiving them. In other words, one may not choose to claim UNB by voluntarily ending one's current employment. However, if one has already suffered loss of employment (involuntarily) and has satisfied the contribution conditions, he is modelled as being able thereafter to hide a possible preference not to work at his prevailing wage. In the absence of necessary National Insurance contributory records and though recognising the

important dynamic considerations involved in any decision to remain unemployed, these working assumptions appear to be the best ones we can devise. It is important to stress, however, that very few of the empirical results of this thesis would be affected by a change in these hypotheses, which are of use only in those circumstances where we wish to illustrate the effect of a change in the observed labour supply behaviour of an agent.

(c) National Insurance Basic Retirement Pension (BP)

Eligibility to BP requires that the recipient be of retirement age (65 for man, 60 for woman) or above and also depends on National Insurance contribution records, of which we have, again, none. BP is also subject to a retirement condition (i.e., $h \leq 12$) condition, after which earning rules apply⁴. To make this module operational, we assume that pension payments other than BP are received unconditionally as declared, and we suppose that all pensionable heads of income units fulfil the NI contributory condition for eligibility to BP⁵; a married woman will, however, receive only the increase for a dependant wife or child-minder. BP then represents a sizable proportion of 35% of all benefits modelled in 1985 Britain, and no less than 7.0% of total gross income.

(d) The Personal Income Tax

We start by applying the 1985 rates of tax (which vary from a basic rate of 30% to a maximum rate of 60%) on the relevant taxable incomes (made of labour

⁴ This rule has by now been removed. Again, see Appendix B for a brief survey of the changes in the British tax and benefit system between 1985 and 1992.

⁵ All reasonable efforts have, however, been made to screen the sample and to identify those for whom it was clearly possible to conclude that NI contributory conditions were not met.

income from main and secondary employments, retirement pensions, unemployment benefits, investment incomes, statutory sick pay, etc.). This first exercise also incorporates the personal allowances (single, married, wife's earned income and age allowances) set against such taxable income. The resulting budget constraint may be termed that derived from applying the "elementary" tax rules. Computed taxes obtained from the "elementary" tax rules would exceed by 11.4% those calculated from the net income tax regime of our model. We then add the effect created by other tax provisions. These other provisions will either add to or decrease the "elementary" tax burden by shifting the loci of the tax kinks, changing the implicit marginal tax rates, or affecting the virtual income levels.

Four provisions are considered, and for each an independent set of (h_i, t_i, YV_i) is constructed to show their effect on the global budget constraint and on the income tax system. First is the reduction (and ultimately the elimination) after a given income limit of the age allowance granted to a person if he or his wife is 65 years or older. Second, we analyse the effect of the controversial MITR (Mortgage Interest Tax Relief) provision, which enables most owner-occupiers to deduct from their taxable income interest payments on mortgages up to £30,000⁶. Furthermore, borrowers not liable to pay tax on their income yet benefit from the relief at the basic tax rate. The relief is thus equivalent to a refundable tax credit whose value will increase with the individual's marginal income tax rate. It also implicitly extends the benefits of the tax exemption of the returns to owner-occupied housing to all those who may have less than £30,000 of savings. Third

⁶ This is still a prominent feature of the current personal income tax system, although limited to the basic rate of tax from 1991-92.

is the deductibility of half of the class 4 National Insurance contributions, an empirically minor provision affecting the self-employed workers. Fourth, we look at the effect of the "composite rate scheme", through which recognised banks and many other deposit-takers (including Building Societies) ought to deduct taxes at a composite rate (slightly below the basic rate of 30%) on payments of interests and relevant dividends. Though they will be set against the overall tax liability of the individual, such basic rate tax payments cannot be refunded if the final tax liability is found to be smaller than the "composite rate" tax originally withheld. This makes this fourth provision a fully regressive one when concatenated onto the "elementary" income tax regime.

The presence of a spouse will affect the overall tax burden of the income unit: the 1985 British income tax system is neither a truly joint nor a separate taxation regime. The division of total income among two spouses will matter for two main reasons: the presence of the wife's non-transferable earned income relief⁷, and the availability for high income units of an embryo of a separate taxation regime⁸.

When building the income tax module in the presence of a spouse, we apply the procedure described above (i.e., the "elementary" and the four other tax provisions) on all of the income unit taxable income. The members of the income unit may also choose separate taxation of the wife's earned income; in such a case, the husband loses all of the excess (£1,450 in 1985) of the married allowance over

⁷ Equivalent in value to the normal single allowance.

⁸ Even if separate taxation is elected, all of the wife's non-earned income (for instance, all of her capital income) and much of the taxable benefits granted to her are taxed as her husband's income.

the single one, but the wife's earned income may then be set against lower marginal tax rates. We thus construct along the joint taxation schedule one under which the spouses are taxed "separately" and finds the point at which a switch from the first schedule to the second is optimal. From this, we may derive the independent effect which the separate taxation option will have on the budget constraint of the income unit.

(e) National Insurance Contributions (NIC)

All workers below pensionable age are liable to pay NIC in addition to the income taxes described in the previous section. We apply to all self-employed workers the "flat-rate" and the Class 2 (6.3%) rate of contributions, and to all others the Class 1 ("not contracted out") regime, which specifies a 9% rate of contribution on all earned income⁹, until an upper earned income limit is reached at which point no more contributions are payable. This, we shall see, makes the contributions first progressive and then regressive, and thus a source of additional nonconvexities in the feasible budget set. Since the earned incomes of two spouses are separately assessed under the NIC provisions, these are an additional source of "independent" taxation. NIC contributions account for 26.2% of all taxes paid in the model. We do not therefore allow for contracting out of NIC, an opportunity which is not straightforward to model using the information available in the FES. It could in any case be argued that those contracted out of the NIC regime would be paying comparable contributions to occupational schemes.

⁹ As long as it is above a lower earned income limit; this thus creates a "nonconvexity" in that net income falls discretely at the specified earned income threshold. The income base also includes sick pay.

(f) Family Income Supplement (FIS, now Family Credit)

This programme aims at supplementing the income of those units with children and of which at least one of the spouses works 30 weekly hours or more (24 in the case of single-parent families). The income base on which the supplement is assessed is made of most sources of incomes and includes UNB and BP when received, but does not take into account income taxes or NIC paid and CB and OPB received. When combined with the effect of other modules, this last feature makes possible overall implicit marginal tax rates exceeding one. The empirical importance of FIS is, however, small: in our model, it amounts to less than 0.4% of all benefits payable.

(g) Supplementary Benefits (SB, now Income Support)

This is the programme most directed to the non-working poor (those with sufficiently low incomes and whose insufficient weekly hours of work would prevent them from claiming FIS). Its marginal rate of withdrawal equals 1, but since the income base is made of income net of NIC and income tax (but includes BP and UNB) global implicit tax rates exceeding one will not occur in the hours intervals for which SB is available. In fact, because the availability of SB smooths the net income function by absorbing the irregularities of the taxes which enter SB's income base, it will in many cases level out nonconvexities introduced by the income tax and the NI contributions. This occurs, for instance, when passing through the NIC's lower earnings limit would increase by a discrete amount the size of NIC payable; a SB recipient would, however, receive back this discretely greater amount of contributions in the form of greater SB payments, thus preventing any fall in the level of net income enjoyed.

12% of all transfers are paid in the form of SB. The size of the benefit depends on whether the recipients are deemed to be entitled to long-term (for people above the age of 59 or for those disabled in specified ways) or short-term requirements, and on the characteristics of the income unit. Additional requirements (and thus greater SB benefits) are also granted to those with housing requirements not covered by Housing Benefit [see section (h)]: these additional entitlements strictly cover the mortgage costs incurred by the poorer owner-occupiers.

(h) Housing Benefits (HB) and Supplementary Housing Benefits

(HBS)

HB and HBS are means-tested benefits aimed at alleviating the burden of rents and domestic rates. Different HB rules apply according to whether the income unit is in receipt of supplementary benefits, and HBS may supplement the effect of HB when a "means-test" prevents the income unit from claiming supplementary benefits. The income for which the Housing Benefit is assessed includes BP, UNB and FIS, and is gross of income taxes and NIC paid.

In conjunction with MITR and the supplementary benefits granted in respect of housing requirements not covered by HB, HB and HBS summarise here the extent to which the availability of state support to an income unit depends on the unit's housing choice¹⁰. Such a dependence may be a clear source of "horizontal inequity", a matter with which we will concern ourselves in Chapter

¹⁰ The analysis of housing finance is very limited in this study. For example, general housing subsidies and capital grants to ease net housing costs are not included, just as we have not encompassed here the level of implicit subsidies arising from the non-taxation of owner-occupiers' imputed rent and capital gains. For more on this, see Hills (1991a).

II. Furthermore, "housing-dependent" benefits jointly constitute a potentially distortive subsidy to housing that amounts in our model to no less than 6% of total gross income. The only ones not in receipt of such subsidy are those owner-occupiers without mortgages (whose returns to ownership are nevertheless tax-exempt and are thus implicitly subsidised) and not entitled to rate rebates, and those in rented accommodation but whose larger income prevents them from claiming HB and HBS.

B- Budget Sets and the Tax and Benefit System

1- The 1985 Budget Sets

Tables 1.3 and 1.4 illustrate the rules incorporated in the programmes briefly described above. In each table, we find under column h the hour values of the upper end of tax and benefit intervals. t shows the tax and withdrawal rates applicable to each of the interval. y^* is the level of virtual income generated by the tax and benefit system and shown above to enter the definition of YV. T is the level of taxes ("minus the amount of benefits") payable for each h or interval end indicated. Table 1.3 thus shows the matrices $[(h_{j,i}, t_{j,i}, YV_{j,i})]$ for each of the i 's pertaining to the income tax regime of an owner-occupier family of four, and also the values of $T(wh)$ yielded by such vectors at the upper kinks $h_{j,i}$. Table 1.4 shows corresponding matrices for aggregates of the programmes described above and for a typical renting family of four. These relationships are also portrayed graphically on Figures 1.1 and 1.2, respectively. In each case, variations in hourly wages would (in most cases) shift proportionately the locus of the hours of work denoting the interval ends.

We see on Table 1.3 that, for instance, an income tax rate of 30% applies to hours of work between 11.39 and 17.47 and, again, between 18.57 and 80.41; the presence of the zero income tax rate between hours of work 17.47 and 18.57 is, as we shall see, a feature of the Composite Rate scheme. The "elementary" marginal income tax rate increases from 0 to 0.45 in the range of hours of work considered. The presence of MITR introduces the equivalent of a lump-sum refundable tax credit of £8.91 for hours of work lower than 74.83 -- or until the value of the mortgage interest relief starts to increase when the tax payer enters tax brackets with higher applicable marginal tax rates. MITR also shifts out the location of the kinks of the elementary income tax (e.g., from $h=74.33$ to $h=80.41$, for the beginning of the 0.4 marginal tax band). Basic rate tax paid at source on interest and relevant dividends under the composite rate scheme is not refundable -- even when the interest or dividend recipient should be a non-tax payer -- and is therefore highly regressive. Such taxes paid at source can, however, be set to reduce those payable under the global income tax schedule. Between $h=17.47$ and $h=18.57$ in Table 1.3, no more income tax is due when labour income increases than the amount already paid under the composite rate scheme. This decreases the overall income tax rate on labour income from 0.3 to 0, thus inducing a possible nonconvexity in the budget set.

At low gross incomes, the net income tax effect on net income may be positive, due to the refundable MITR tax credit, unless this is outweighed by the negative effect of the non-refundable tax withheld by the composite rate scheme. In all cases, it is clear from the above tables and figures that -- particularly so for low incomes and labour supply -- the elementary income tax

system (of which the rules are probably among the easiest and most widely known of the tax and transfer system) does not approximate well the true income tax schedules, not to mention the complete tax and transfer regime. It also appears that the net income tax regime may well be less progressive than that which the elementary rules would suggest. Finally, a quick look at the overall budget constraint of Table 1.4 and Figure 1.2 suggests a large variability of the marginal tax rates and virtual incomes (of which y^* is a component), due to the intricate combination of tax and benefit provisions and to the usual progressivity of the various programmes.

Figure 1.3 displays some of the variability of net budget sets in the dimension of income unit characteristics. It must be noted that these income unit characteristics also include varying wage rates, as displayed in Table 1.1. All income units (apart from the pensionable couple) face SB-imposed implicit marginal tax rates of one at low levels of hours of work. This would effectively discourage any low-hours, part-time jobs. Given the units' typical characteristics (e.g., their wage level and SB requirements), it can be seen in Figure 1.3 that the average pensionable couple would be better off (ignoring possible differences in needs) than other average income unit types for husbands' hours of work below 30. The average "accommodation-renting" family of four would conversely (for hours up to 50) be worse off than all others.

Figures 1.4 and 1.5 shed some light on the important issue of work incentives. Budget sets based on the average wage of three hourly wage quintiles of families of four (tenants) are exhibited in Figure 1.4. The budget constraint of the lowest quintile displays by its particularly shallow slope the potential for a

poverty trap induced by the tax and transfer system: indeed, between $h=30$ (where a switch from SB to FIS causes a discrete shift upward of the budget constraint) and $h=44$, the combination of the marginal tax rates of personal income taxation (30%), NIC (9%), FIS (50%) and HB (19%) makes net income decrease continuously with hours of work.

The information shown on Figure 1.4 can also be portrayed on a three-dimensional graph: this is essentially done in Figure 1.5, and exposes the variation of net income as a function of the gross wage and hours of work. The resulting picture is one of cliffs, increasingly and decreasingly steep hill slopes, and plateaux¹¹. The initial upward slope starting at zero hours of work flows from the deductibility of the first £4 of earnings of those on part-time work who wish to receive Supplementary Benefits. The availability of Supplementary Benefits, whose amount decreases one-for-one with the amount of family earnings, subsequently flattens the distribution of net income at about 95 pounds per week for low wages or low hours of work, and thus creates the lower plateau. When entitlement to Supplementary Benefits ends, net income starts to rise with hours of work and the hourly wage rate, but the slope varies with the withdrawal rates imposed on housing benefits and with the perception rates of National Insurance Contributions and income taxes. The cliff at 30 hours of work (and for wage rates up to about £3.22) is induced by the Family Income Supplement's availability limited to those working more than 30 hours a week (the benefit is also means-

¹¹ An updated graph for the 1991 tax and benefit system -- along with a discussion of how the depiction of the system would be affected by the replacement of the various tax and benefit provisions by a "basic" or "citizen's" income -- can be found in Duclos (1992a). The discussion there includes most of the tax and benefit changes that have occurred since 1985 and which are summarised in Appendix B.

tested). Implicit tax rates vary from 39% to 108%. This latter rate causes anomalies that transcends the distribution of hours of work: for instance, a head working 30 hours at a wage of £2.19 per hour would have a net weekly income greater (by £2.50) than that of a head providing the same number of hours but at a wage of £3.22; such features of real tax and benefit systems clearly raise questions about the fairness or equity of the system, which we discuss in the next chapter.

2- The Distribution of Marginal Tax Rates

Tables 1.5a and 1.6a indicate for each labour income decile group the distribution of implicit and explicit marginal tax rates for the head and the wife (if there is one) of the income units, respectively. "Heads" are as defined in the FES, and thus refer to the man in the presence of a couple; income units are also as defined in the FES and will thus include many of the young adults living at home. Besides illustrating the structure of the tax and benefit system, tables displaying such marginal tax rates yield information on the marginal return to increased savings and investment and to greater labour market involvement (more hours of work or enhanced productivity and hourly wages). It would, however, be unwise to draw any strong incentive conclusions from statistics of this kind since work and savings behaviour often depends much on the average return (e.g., where the choice is between working full-time or not at all) and on the distribution of income unit characteristics (great grand-fathers, who earn no labour income and feature at the bottom of the labour income distribution, would probably earn no more even if their implicit and explicit marginal tax rates were to be decreased).

The information provided by Tables 1.5a and 1.6a reinforces some of the

comments made earlier. Personal income tax is a bad indicator of the net budget sets; in fact, marginal explicit income tax rates are strongly negatively correlated with the net implicit tax rate over much of the decile distribution. Furthermore, for heads of income units, this net rate *falls monotonically* with labour income. Notwithstanding the comments just made in the previous paragraph, personal tax reform proposals explicitly concerned with work incentives could thus easily spring from a misinterpretation of the true distribution of marginal tax rates. Table 1.6a also suggests that wives face significantly lower marginal tax rates than their husbands' if, as is commonly assumed, their marginal labour supply decisions are made taking as given the labour supply of their spouse. Unlike those of Table 1.5a, net marginal tax rates do not fall monotonically, in line with the more predominant role of the marginal income tax rates.

How would these results, and the results derived from our model generally, be affected by a correction for departures of our sample from population characteristics? As indicated briefly in the "Statistical Notes on the Use of the 1985 Family Expenditure Survey Data" (Appendix A) the FES data may fail to represent accurately a micro picture of the true population due to, for instance, sampling practices and differential response rates among those income units selected. Atkinson, Gomulka and Sutherland (1988) attempted to lessen the effects of such irregularities by using UK population data on household composition, income ranges, housing tenure, age, employment status and regions to compute "grossing-up weights" that vary with income unit characteristics. These weights are positively correlated with the status of a pensioner, single person, owner-occupier, childless household, and an income in the upper ranges, reflecting the

suspected under-representation of such groups in the FES sample.

We first note that the administrative aggregates amount to very close to our grossed-up results for those benefits which are most straightforwardly modelled. This can be checked in Table 1.7, where the grossed-up number of recipients and the grossed-up value of the receipts of CB are quite close to those registered in the Department of Social Security official statistics. Recipients and receipts of OPB differ rather more, as cohabitation rules must in principle be assessed before that benefit can be granted. The grossed figure of FIS receipts and recipients show surprising (and perhaps worrying) coincidence given the difficulties involved in evaluating FIS entitlement from the information found in the FES. We are only able to model *current* entitlement whereas actual receipt reflects eligibility at earlier dates. It must moreover be remembered that our grossed-up results in Table 1.7 assume full take-up of the benefits and that this necessarily favours the overestimation -- and a possibly sizeable one -- of the numbers that would have otherwise been predicted. Finally, and as we shall discuss much more in Chapter IV, aggregate FES figures of SB receipts and recipients look rather imprecise, especially if we were to reduce them by the incomplete take-up rates reported in previous studies (more on this in Chapter IV).

Tables 1.5b and 1.6b then indicate the distribution of marginal tax rates when such varying grossing-up weights are used, instead of the implicitly equal weights used in Tables 1.5a and 1.6a. The most significant changes occur in the lower labour income deciles, for which the increased proportions of childless, pensioner and one-adult units contribute to a fall in the average income support withdrawal rate, and subsequently to a fall in the net implicit marginal tax rate.

By indicating the distribution of marginal tax rates of husbands when the full labour income of the spouse is recognised, Table 1.5d presents a better perspective of the labour choice made by income units at the margin and by husbands taking as given the labour supply of their wives. Moving from Tables 1.5d to 1.5c, where the spouse is assumed to have no labour income, we note a tightening of marginal rates, effected by an increase in the implicit withdrawal rates of benefits but attenuated slightly by a fall in the marginal tax rates. Husbands who recognise the impact of their spouse's labour market behaviour upon their own tax and benefit schedule thus witness an increase in their income tax rate but also a substantial fall in their benefit withdrawal rate, the net result on their final implicit tax rate depending on the size of their labour income and other resources.

Comparing Tables 1.6a and 1.6c shows the effect of reversing the assumption previously made on the wife's perception of the labour income of her spouse. Bringing the husband's labour supply to zero greatly increases the wife's marginal benefit withdrawal rates but does not simultaneously decrease so much the marginal rate at which income taxes are paid. The net result is that the net marginal rates of Table 1.6c exceed everywhere those of Table 1.6a, where wives take into account their husband's earned income in assessing the marginal return to work. This runs contrary to the sometimes expressed view that the husband's work increases applicable marginal tax rates on the wife's labour income and thus decreases her incentive to work.

As suggested above, however, average tax rates on earned income may yield a better guide to work incentives than marginal ones; moreover, the debate

on respective work incentives often revolves around the relative size of the husbands' and wives' marginal tax rates. Comparing Tables 1.5d and 1.6a, where both husbands and wives take full account of their spouse's earnings in assessing marginal tax rates, we note -- at the observed level of labour supply -- that the marginal tax and benefit incentive to the wife is relatively better than that to the husband. This holds with the exception of those in full-time work or with high earnings, where the joint taxation of income ensures that husbands and wives face broadly similar net marginal rates. The wives' separate earnings allowance grants to wives a degree of relief from income tax at low earnings which is not similarly given to husbands; this also guarantees that income tax rates for wives in Table 1.6a are always no greater than those for their husbands in Table 1.5d.

3- Nonconvexities of the Budget Set

The presence of taxes and benefits may also introduce significant nonconvexities in the shape of the budget sets, examples of which are shown in Figure 1.6a. There are three types of such non-convexities, and all three are created by the tax and benefit regulations of the 1985 UK system:

- (1) An increase in the net wage, produced by a fall in the net marginal tax rate between two linear segments. A similar nonconvexity would be generated by a gross or market wage that increased with hours supplied, which is empirically plausible.
- (2) A discrete fall in net income, produced, say, by a sudden fall of entitlement to some benefit or by taxes (such as NIC) being levied suddenly on a discrete level of income or capital. This could further induce reranking among units situated on either side of the discrete fall in net

income. Equivalently, as in Hausman (1980), falls in net income may occur at different levels of hours of work if there are fixed costs to participation in the labour market.

(3) A discrete increase in net income: this can easily arise from the dependency of the entitlement to some benefit on working a minimum level of hours (such as for FIS).

These nonconvexities can lead, among other things, to multiple tangencies of indifference curves with the non-concave budget curve and to large changes in desired labour supply following relatively minor disturbances to the budget constraint. Their presence also complicates sizeably the estimation and the interpretation of labour supply curves, as discussed for instance in Hausman (1985), Blomquist (1989) and Duclos (1990).

Such apparent irregularities may not, however, always be suboptimal. As discussed by Blinder and Rosen (1985), discrete falls or increases in the level of taxes or subsidies can yield more desirable outcomes than a continuous form of subsidisation or taxation that induces welfare costs (or deadweight losses) over a potentially wide interval of hours of work. Such "notches" will nevertheless generate sizeable welfare costs for those (but only for those) who will end up consuming or selling at the induced kinks. It then follows that "the aggregate excess burden depends on the distribution of individual tastes" [Blinder and Rosen (1985), p.738]. Blinder and Rosen then simulate the welfare and budget impact of favouring the consumption of a preferred good through the application of a standard price subsidy and of a notch one -- i.e., agents receive some lump-sum income if and only if they choose to consume more than a given quantity of

the favoured good. Under the constraint of achieving a minimum aggregate consumption level of the favoured good, notches can, when properly designed, "induce individuals to self-select so that those who are most willing to change their behaviour are the ones who receive the subsidy (or avoid the tax)" [Blinder and Rosen (1985), p.745]. Deadweight losses are then kept under control and no price subsidies are wasted upon those little willing to increase (or decrease) their consumption of the preferred (or undesirable) good.

Some mental gymnastics is needed to adapt such insights to a world in which the desired achievement of some aggregate consumption is supplanted by considerations of income distribution. Figure 1.7 helps portray the issues involved. On both parts of the figure we find a graph of net against gross income (gross income being equal to earnings, for simplicity). On the top part of Figure 1.7 we find the presence of a negative income tax (NIT) which is terminated by an abrupt downwards notch at wh_0 ; the programme is described by the line segments ACB . Units with conventional indifference curves U_i (such as U_0 and U_1) would not choose to work between points B and D but would rather remain at a point between A and C and claim a level of benefits equal to the horizontal distance between such a point and the gross income (45°) line. Other units with a lower relative utility of leisure could also wish to earn more than at D , but they would not typically choose hours of work between B and D .

The notch at C thus perversely discourages some of the poor from earning more and keeps them dependent on state support. Filling the benefit gap between C and E by instituting a full NIT between A and E and hence by removing the downward notch worsens no one's situation and could even possibly be done for

a lower level of aggregate government expenditures. Removing the downwards notch improves the lot of those formerly at the kink C , who can then relocate to F , with $U_1 > U_0$, with reduced benefit dependency and thus with a lower level of state expenditures. Some of those who used to be to the right of D and would not have been recipient of state support could also gain, potentially cutting down their level of labour supply and earnings and moving onto the segment CE , where their greater level of welfare springs from their receipt of state support. Whether aggregate government expenditures are lowered, unchanged or increased by such a policy change therefore depends on the relative size of these two groups of people and on the magnitude of their adjustment to the extended NIT.

Removing downward notches can thus be optimal, and will be necessarily so if the additional government expenditures incurred on those who reduce their earnings to the segment CE do not exceed the level of government savings achieved from the lesser benefit dependency of those who increase their labour supply from C to above it. Conventional social welfare concerns would then also be satisfied by such a policy change. Our argument is of course at best heuristic, and a proper treatment would require a full mathematical formulation or numerical simulations using some empirical labour supply estimates, which is outside the scope of this paper.

Could it also be better to go even more forward and generate upward notches? The bottom part of Figure 1.7 helps grasp the issues then involved. As before, we find the 45° gross income line in the *net income, gross earnings* space, but this time along with a dashed line KL depicting the effect of the SB (or poverty alleviation) regime found in Britain and a dotted one $GHIJ$ showing the impact of

the institution of a notch subsidy (of the FIS or "workfare" type) for those earning more than wh_1 (at point H). As we shall see presently, the construction of these two lines can make them cost the same in aggregate expenditures to the state; for simplicity, also suppose that the distance IH is smaller than the one from the origin to K . Those agents with indifference curves U_2 and U_3 clearly lose from the move to the notch subsidy: their net income and benefit receipt falls from K to G , with no change in their gross earnings. Those agents with the indifference curve U_4 could lose in welfare from the switch to the dotted line but can also avoid this by moving from K to I . The level of benefits granted to such individuals also falls. For some with an even greater taste for work but who also are at K initially, utility will increase but receipt of state support will nevertheless also fall. For all those on the segment LJ initially, welfare will also increase but the state will then have to incur for them some additional subsidy expenditures. For a given distribution of utility functions, a suitable choice of the size of wh_1 and of the length of HI could make the workfare programme $GHIJ$ demand the same level of government expenditures as that required by the SB-type programme KL .

It is clear from the above that even if aggregate deadweight losses were to fall (which is not guaranteed) after the switch to the notch subsidy for constant aggregate state transfers, the desirability of the reform would depend strongly on the distributional tastes of the state. It may be a matter of great concern that those ultimately at G will suffer from the switch and will be both absolutely and relatively worse off than with the SB type scheme. Alternatively, for political and efficiency reasons, the FIS or workfare scheme may prevail since it can make the redistribution of income towards at least some of the original poor more

acceptable. Such arguments are also discussed theoretically in, for instance, Fortin et al.(1990). There, simulation results provided for the Québec economy also suggest that a categorical **combination** of workfare and NIT programmes could optimally be devised, where such programmes would be made available in the spirit of Akerlof (1978) on the basis of observable categorical characteristics. Again, we must limit our own analysis to the above intuitive considerations, although the topic is clearly one worthy of a more detailed analytical and empirical enquiry.

Figures 1.8 and 1.9 throw some light on the empirical presence of such nonconvexities in the 1985 budget sets of those heads of income units with wages belonging to the middle quintile group. That is, we order hourly wages of the heads from the lowest to the highest, pick up the 20% in the middle, and vary the hours of work of each of these heads from 0 to 80. We then compute for each head the number of nonconvexities occurring in each of the 80 hourly ranges and express this number as an average number of nonconvexities in each hourly range over this subsample of heads of units.

Figure 1.8 indicates that, for these agents, most of the nonconvexities occur between $h=0$ and $h=30$, and come foremost from the NIC, Supplementary Benefits, Family Income Supplement and housing benefits rules. There is a discrete fall in net income at around 12 hours a week when agents start to pay NIC contributions on all of their labour income¹². The ending of payments of further National Insurance Contributions also creates an additional nonconvexity at above $h=80$ for

¹² This NIC feature has now been slightly altered, such that workers are liable to pay only up to 2% of their earnings below the Lower Earnings Limit, thus limiting the size of the nonconvexity and of the fall in net income.

the third wage quintile. Jumps in the level of net income occur mostly below $h=24$, at the start of the applicability of Supplementary Housing Benefits, and at 24 and 30 hours of work a week, where support from the state is shifted from Supplementary Benefits (now Income Support) to the normally more generous Family Income Supplement programme (now Family Credit). Housing benefits introduce nonconvexities throughout the hour range, mostly in the form of falls in the marginal implicit tax rates on labour income.

Figure 1.9 shows the type of the nonconvexities introduced in the net budget sets: that is, whether they stem from an increase in the net wage, or from a discrete fall or jump in net income. We note that most of them are in the form of increases in the marginal wage (from falls in the marginal net tax rate), and that such rises in the marginal return to working occur over the whole range of hours. The mode of the distribution of the discrete falls in net income is 12 hours, where NIC start to be payable, and peaks in the occurrence of upward notches in the level of net income take place at 24 and 30 hours, where entitlement to FIS becomes possible.

4- The Elasticity of the Virtual Wage

How sensitive is the *net* wage to changes in the *gross* wage or to changes in the supply of labour? In other words, how good an approximation of the budget set would linearisations of the budget constraint around the level of actual weekly hours of work h provide? To answer such questions we may compute the elasticity of the virtual (or net) wage with respect to the gross wage when hours of work are kept constant. This is defined as:

$$\epsilon_{w_v, w} \equiv \frac{d(w_v)}{dw} \cdot \frac{w}{w_v} \quad (7)$$

with $w_v = (1-t)w$ and $t = \sum t_i$.

In the presence of m taxes and benefits, we can decompose this into:

$$\epsilon_{w_v, w} = 1 + \sum_{i=1}^m \left[\left(\frac{d(1-t_i)}{dw} \cdot \frac{w}{(1-t_i)} \cdot \right) \frac{1-t_i}{(1-\sum t_i)} \right] \quad (8)$$

which is equivalent to a weighted sum of the individual $1-t_i$ elasticities:

$$\epsilon_{w_v, w} = 1 + \left[\sum_{i=1}^m \epsilon_{(1-t_i)w} \frac{1-t_i}{1-\sum t_i} \right] \quad (9)$$

Because the British tax and benefit system is essentially piece-wise linear at any point of the budget curve, this elasticity of the net wage with respect to the gross wage will typically be one, except of course at kink points where discrete changes in the net tax rate would yield infinite absolute values to the elasticity. This feature of the British system suggests that it may be more fruitful to assess the average elasticity of the marginal wage over a finite variation in the level of gross wages – that is, by proportionately how much, on average, would the marginal wage w_v of units change if their market wage were to vary by a given percentage, keeping the units' labour supply constant? Measuring in this fashion the impact of discrete percentage changes in the level of the gross wage implicitly implies the approximation of the potentially distinct linear tax and benefit segments over

which an agent would move by only one continuously differentiable curve: finding, for instance, an elasticity of 10% for a 25% change in the level of the gross wage would indicate that the marginal wage would have changed by an average of 2.5% over the width of gross wage variation.

Tables 1.8 and 1.9 thus list the average of the square bracket elements of equation (8) for heads and wives of income units when $\Delta w/w$ varies from 0.01 to 0.25. These sum to the elasticity of the net wage with respect to hours supplied, to gross income, or, if added to 1, to the gross wage. In parentheses we show the proportion of individuals whose marginal wage remains unaffected by the proportional increase in the gross or market wage.

Two comments are first in order. First, decomposing the elasticity of the virtual wage as done above should reasonably require that taxes and benefits be additively separable, a feature which the 1985 British system does not possess. It is the case, for instance, that the level of eligibility to HB depends on the level of income net of FIS; implicit FIS tax rates are thus also felt through their impact on the HB marginal tax rates. Second, the results presented in Tables 1.8 and 1.9 are limited to **averages** of income unit elasticities whose distribution and variance across the sample of heads of units is very wide. Many heads and wives notice no change in their marginal tax rate as their labour supply or gross wage is proportionately increased, in part because many of these agents are not part of the labour force. An increase of 25% of everyone's labour supply will, for instance, change the virtual wage of fewer than 22% of the heads and 14% of the wives.

The income tax and NIC system generally contributes negatively to the overall elasticity, whereas the benefit side does so positively. This is

understandable: as we saw earlier, moving up the earned income scale usually meant an increase in the average marginal income tax rate (a fall in the virtual wage) but a fall in the applicable benefit withdrawal rates (an increase in the virtual wage). The impact of the income tax system being relatively more significant on the wife's budget constraint, the wives' overall virtual wage elasticity thus ends up below one while that of their spouse is significantly above one for all checks of $\Delta w/w$. That is, a proportional change in the gross wage of the head (wife) will at the margin more (less) than proportionally increase the virtual wage faced by the individual. We also note that various taxes and benefits have various impacts upon the final marginal wage elasticity. Housing Benefits, however more limited in scope than SB and NIC for instance, exert a greater influence on the degree of non-linearity of the heads' budget curve than all other tax and benefit components. From Table 1.8, we thus find that a 1% increase in heads' gross wages will, at the average elasticity, increase the marginal wage by 0.26%, from a sizeable positive HB contribution (withdrawal rates have fallen), a negligible NIC contribution, and a negative income tax element (marginal income tax rates are increasing).

Since the knowledge of the virtual wage elasticities could give us a crude appraisal of differential labour supply adjustments following changes in the gross wage or in the amount of gross income, we consider in Figures 1.9 and 1.10 the distribution of such elasticities (the elements in the squared brackets of the virtual wage elasticity equation) across labour income decile groups. For the 43% of heads and 52% of wives who did not work, such elasticities are always zero: no change in the marginal tax rates follows from a % change in their wage or labour

supply level. Figure 1.10 indicates that, as we move from the 5th to the highest labour income decile of the heads, $\epsilon_{1-l,w}$ increases from a low level ($\epsilon_{wv,w}$ could even be negative if marginal tax rates increased sufficiently rapidly as w rises) as heads first make their way through and then leave the steep withdrawal rates of the transfer system. NIC first drives $\epsilon_{wv,w}$ down as contributions start being deducted, but keep $\epsilon_{wv,w}$ above one once the upper earned income limit for the payment of contributions is reached.

Wives (Figure 1.11) face a relationship inverse to that of the heads, for reasons mentioned above. As we move up the income deciles, the virtual wage first increases less than proportionally, then more so, following changes in the gross wage. All this also suggests that uniform labour supply adjustments to uniform wage shocks would be ruled out even in the presence of homogeneous elasticities of labour supply with respect to income and net wages.

Conclusion

We recall that this chapter served as a stepping stone to the empirical applications offered in the rest of this thesis. We first describe rather succinctly the data and the structure of our underlying computer model; we then strive to illustrate the features of the 1985 British tax and benefit system. To this end, we portray the system's associated budget sets, we note the distribution of marginal tax rates, we discuss the concordance of our grossed-up statistics with administrative ones, we analyse the elasticity of the net wage with respect to changes in pre-tax earnings, and we disentangle the distribution of non-convexities and notches in the tax and benefit system.

Our main conclusions in this chapter are mostly illustrative. We saw that

what is widely regarded as the tax and benefit system's most prominent feature -- the personal income tax -- describes poorly the shape of the budget sets available to family units. Besides displaying "cliffs" and "plateaux", the 1985 British system exhibits a wide variation of marginal tax rates that generally decrease with labour income; these are usual features of systems where the targeting of benefits can only induce sizeable "poverty" and "unemployment traps". This also explains why wives of working husbands typically face lower marginal tax rates than if the husbands earned less or not at all. The withdrawal and substitution of benefits also creates numerous budget "nonconvexities", which occur mostly at lower levels of earnings and hours of work. We have also seen how these can be justifiable or unjustifiable in the context of an optimal tax or benefit design. A small change in hours of work, gross income, or the wage rate will usually not change the net wage to additional work effort, though if it does the effect will be very significant. Plotting the average of the net wage elasticities confirms the earlier observations that the net wage is a stable proportion of the gross wage only at relatively large levels of earnings, where the withdrawal of state benefits has ended.

Table 1.1

Characteristics of Some Typical Income Units ¹

Characteristic	(1)	(2)	(3)	(4)
Age	38	36	69	36
Age of Spouse	36	33	68	34
Personal Allowance	65.6	65.6	80.8	65.6
Wife's Allowance	41.9	41.9	41.9	41.9
Composite Tax Retained	1.62	0.21	5.52	0.3
Basic Pension Eligibility	0	0	57.3	0
Eligible Rates and Rents	7.18	24.0	6.21	5.7
Supplementary Benefits Requirements	94.5	64.8	58.4	88.5
Hourly Wage	4.89	3.56	2.07	3.95
Hourly Wage of Spouse	2.70	2.33	1.86	2.42
Mortgage Interest Payments	29.7	0	1.26	23.8
Non-labour Taxable Income	9.95	2.24	55.23	6.25
Non-labour Income	30.2	30.1	59.35	35.6

(1) family of four, owner-occupiers;

(2) family of four, renters;

(3) pensioner couple, owner-occupier;

(4) family of four, of which the head is unemployed, owner-occupiers.

¹ All monetary variables are in £ per week.

Table 1.2

Tax and Benefit Modelling

Sequence of income and modelled taxes and benefits	Income base for assessment of taxes and benefits include:
Original income ²	
Various observed benefits	
(a) CB and OPB	not income-dependent
(b) UNB	earned income
(c) BP	earned income
(d) Personal income tax	UNB and BP (not CB and OPB)
(e) NIC	labour income
(f) FIS	UNB, BP (not CB, OPB, personal income tax and NIC)
(g) SB	CB, OPB, BP, UNB, personal income tax, NIC
(h) HB, HBS	CB, OPB, BP, UNB, FIS (but not personal income tax and SB)

² The details of "original" or gross income and of "various observed benefits" are shown in Appendix A.

Table 1.3

(h,t,y*) Vectors of the Income Tax System

Family of Four, Owner-Occupiers

h	Income Tax			"Elementary" Y tax			MITR			Composite Rate Scheme		
	t	y*	T	t	y*	T	t	y*	T	t	y*	T
0	0	7.29	-7.29	0	0	0	0	8.91	-8.91	0	-1.62	1.62
11.39	0	7.29	-7.29	0	0	0	0	8.91	-8.91	0	-1.62	1.62
17.47	0.3	24.00	1.62	0.3	16.71	8.92	0	8.91	-8.91	0	-1.62	1.62
18.57	0	-1.62	1.62	0.3	16.71	10.53	0	8.91	-8.91	-0.3	-27.2	0
74.33	0.3	25.62	83.42	0.3	16.71	92.34	0	8.91	-8.91	0	0	0
80.41	0.3	25.62	92.34	0.4	53.06	104.2	-0.1	-27.44	-11.88	0	0	0
85.99	0.4	64.94	103.3	0.4	53.06	115.1	0	11.88	-11.88	0	0	0
92.07	0.4	64.94	115.1	0.45	74.08	128.5	-0.05	-9.14	-13.37	0	0	0

Table 1.4

(h,t,y*) Vectors of the Global Budget Constraint

Family of Four, Renting

h	Cumulative Vectors			Income Tax and NIC			FIS and SB			HB and HBS		
	t	y*	T	t	y*	T	t	y*	T	t	y*	T
0	1.00	60.95	-61.0	0	-0.21	0.21	1.00	39.23	-39.2	0	21.93	-21.9
9.89	1.00	60.95	-25.8	0	-0.21	0.21	1.00	39.23	-4.02	0	21.93	-21.9
12.12	1.00	60.95	-17.8	0.09	-0.24	4.12	0.91	39.26	0	0	21.93	-21.9
12.77	1.00	63.06	-17.6	0.09	-0.24	4.33	0	0	0	0.91	63.30	-21.9
14.02	0.09	21.69	-17.2	0.09	-0.24	4.73	0	0	0	0	21.93	-21.9
14.15	0.17	25.69	-17.1	0.09	-0.24	4.79	0	0	0	0.08	25.92	-21.9
18.01	0.42	38.28	-11.4	0.09	-0.24	6.00	0	0	0	0.33	38.51	-17.4
22.3	0.72	57.51	-0.34	0.39	19.00	11.97	0	0	0	0.33	38.51	-12.3
30.04	0.77	61.48	20.88	0.39	19.00	22.70	0	0	0	0.38	42.48	-1.84
31.82	0.68	51.85	25.19	0.39	19.00	25.18	0	0	0	0.29	32.86	0
74.36	0.39	19.00	84.24	0.39	19.00	84.24	0	0	0	0	0	0
104.3	0.30	-4.83	116.2	0.30	-4.83	116.2	0	0	0	0	0	0

Table 1.5a

Distribution of Implicit and Explicit Marginal Tax Rates

Head of Income Unit³

Deciles of heads' Labour Income ⁴	Marginal Tax Rate					
	Income Tax	NIC	UNB & BP	Income Support	Housing Benefit	Net Rate
1	0.103	0	0	0.376	0.102	0.580
2	0.103	0	0	0.376	0.102	0.580
3	0.103	0	0	0.376	0.102	0.580
4	0.103	0	0	0.376	0.102	0.580
5	0.178	0.053	0	0.133	0.181	0.545
6	0.295	0.088	0.007	0.036	0.090	0.516
7	0.298	0.090	0.002	0.006	0.042	0.438
8	0.302	0.090	0	0	0.005	0.397
9	0.302	0.089	0	0	0.001	0.392
10	0.333	0.035	0	0	0	0.367

³ Under the assumption that the spouse has no labour income.

⁴ Over 43% of the heads of the income units had no labour income.

Table 1.5b

Distribution of Implicit and Explicit Marginal Tax Rates

Head of Income Unit⁵

Varying grossing-up weights

Deciles of heads' Labour Income ⁶	Marginal Tax Rate					
	Income Tax	NIC	UNB & BP	Income Support	Housing Benefit	Net Rate
1	0.102	0	0	0.360	0.102	0.565
2	0.102	0	0	0.360	0.102	0.565
3	0.102	0	0	0.360	0.102	0.565
4	0.102	0	0	0.360	0.102	0.565
5	0.182	0.055	0	0.121	0.183	0.541
6	0.296	0.089	0.006	0.030	0.086	0.506
7	0.299	0.090	0.001	0.009	0.038	0.437
8	0.302	0.090	0	0	0.004	0.396
9	0.302	0.089	0	0	0.001	0.392
10	0.337	0.034	0	0	0	0.371

⁵ Under the assumption that the spouse has no labour income.

⁶ Over 43% of the heads of the income units had no labour income.

Table 1.5c

Distribution of Implicit and Explicit Marginal Tax Rates

Husband⁷

Deciles of husbands' Labour Income ⁸	Marginal Tax Rate					
	Income Tax	NIC	UNB & BP	Income Support	Housing Benefit	Net Rate
1	0.074	0	0	0.487	0.083	0.644
2	0.074	0	0	0.487	0.083	0.644
3	0.074	0	0	0.487	0.083	0.644
4	0.179	0.054	0.007	0.248	0.251	0.739
5	0.296	0.089	0.004	0.033	0.119	0.542
6	0.300	0.090	0	0.004	0.023	0.418
7	0.304	0.090	0	0	0.002	0.395
8	0.302	0.088	0	0	0.002	0.392
9	0.301	0.083	0	0	0	0.384
10	0.361	0	0	0	0	0.361

⁷ Under the assumption that the spouse has no labour income.

⁸ Over 36% of the husbands (i.e., those heads who live with a spouse) of the income units had no labour income.

Table 1.5d

Distribution of Implicit and Explicit Marginal Tax Rates

Husband⁹

Deciles of husbands' Labour Income ¹⁰	Marginal Tax Rate					
	Income Tax	NIC	UNB & BP	Income Support	Housing Benefit	Net Rate
1	0.157	0	0	0.207	0.079	0.443
2	0.157	0	0	0.207	0.079	0.443
3	0.157	0	0	0.207	0.079	0.443
4	0.204	0.054	0.007	0.125	0.165	0.554
5	0.299	0.089	0.004	0.015	0.055	0.463
6	0.302	0.090	0	0.004	0.013	0.409
7	0.304	0.090	0	0	0	0.394
8	0.303	0.088	0	0	0.001	0.392
9	0.308	0.083	0	0	0	0.391
10	0.376	0	0	0	0	0.376

⁹Taking the labour income of the spouse as given.

¹⁰ Over 36% of the husbands (i.e., those heads who live with a spouse) of the income units had no labour income.

Table 1.6a

Distribution of Implicit and Explicit Marginal Tax Rates

Wife's Budget Constraint¹¹

Deciles of wives' Labour Income ¹²	Marginal Tax Rate					
	Income Tax	NIC	UNB & BP	Income Support	Housing Benefit	Net Rate
1	0	0	0	0.145	0.054	0.201
2	0	0	0	0.145	0.054	0.201
3	0	0	0	0.145	0.054	0.201
4	0	0	0	0.145	0.054	0.201
5	0	0	0	0.145	0.054	0.201
6	0	0	0	0.055	0.017	0.072
7	0.022	0.033	0	0.021	0.028	0.104
8	0.284	0.086	0	0.004	0.018	0.393
9	0.295	0.088	0	0.002	0.006	0.391
10	0.312	0.086	0	0	0.001	0.399

¹¹ Taking the labour income of the spouse as given.

¹² 52% of the wives had no labour income.

Table 1.6b

Distribution of Implicit and Explicit Marginal Tax Rates

Wife's Budget Constraint¹³

Varying grossing-up weights

Deciles of wives' Labour Income ¹⁴	Marginal Tax Rate					
	Income Tax	NIC	UNB & BP	Income Support	Housing Benefit	Net Rate
1	0.002	0	0	0.134	0.056	0.193
2	0.002	0	0	0.134	0.056	0.193
3	0.002	0	0	0.134	0.056	0.193
4	0.002	0	0	0.134	0.056	0.193
5	0.002	0	0	0.134	0.056	0.193
6	0	0	0	0.051	0.017	0.068
7	0.021	0.033	0	0.018	0.030	0.102
8	0.287	0.086	0	0.004	0.017	0.393
9	0.292	0.087	0	0.001	0.006	0.387
10	0.313	0.084	0	0	0.002	0.400

¹³ Taking the labour income of the spouse as given.

¹⁴ 52% of the wives had no labour income.

Table 1.6c

Distribution of Implicit and Explicit Marginal Tax Rates

Wife's Budget Constraint¹⁵

Deciles of wives' Labour Income ¹⁶	Marginal Tax Rate					
	Income Tax	NIC	UNB & BP	Income Support	Housing Benefit	Net Rate
1	0	0	0	0.631	0.055	0.687
2	0	0	0	0.631	0.055	0.687
3	0	0	0	0.631	0.055	0.687
4	0	0	0	0.631	0.055	0.687
5	0	0	0	0.631	0.055	0.687
6	0	0	0	0.843	0.041	0.884
7	0.008	0.033	0	0.704	0.076	0.822
8	0.036	0.086	0	0.380	0.212	0.714
9	0.122	0.088	0	0.057	0.169	0.437
10	0.296	0.086	0	0	0.019	0.401

¹⁵ Assuming no earned income from the husband.

¹⁶ 52% of the wives had no labour income.

Table 1.7

Grossing-up Benefit Recipients and Receipts

Aggregate Number of Recipients and Aggregate Value of Receipts

	Child Benefit		One-Parent Benefit		Family Income Supplement		Supplementary Benefits	
	Recipients (10 ³)	Value (10 ⁶ £) ¹⁷	Recipients (10 ³)	Value (10 ⁶ £)	Recipients (10 ³)	Value (10 ⁶ £)	Recipients (10 ³)	Value (10 ⁶ £)
Administrative Official Data¹⁸	6819	4468	582	134	201	130	4390	5810
Model Values¹⁹, Grossed-up	6615	4429	738	163	239	121	4118	3982

¹⁷ Annual benefits, per annum.

¹⁸ Source: Department of Social Security (1990, 1989).

¹⁹ Model values are based on assumption of full take-up of the modelled value of benefits.

Table 1.8

**Elements of the Elasticity of the
Virtual Wage With Respect to the Gross Wage** ²⁰

Head of Unit's Budget Constraint²¹

$\Delta w/w$	Income Tax	NIC	FIS and SB	UNB and BP	Housing Benefits	All Taxes and Benefits
0.01	-0.037	0.015	0	0	0.281	0.264
	(99.9)	(99.8)	(100)	(99.9)	(99.3)	(99.0)
0.05	-0.038	-0.17	0.037	-0.008	0.163	0.135
	(98.7)	(98.4)	(99.7)	(99.9)	(97.9)	(94.5)
0.10	-0.030	0.007	-0.033	-0.020	0.178	0.102
	(97.0)	(96.8)	(99.6)	(99.9)	(95.3)	(89.4)
0.25	-0.028	0.027	0.008	-0.009	0.134	0.133
	(94.1)	(92.3)	(99.1)	(99.9)	(91.3)	(78.6)

²⁰ Table shows $[\epsilon_{(1-t),w} (1-t)/(1-\Sigma t)]$. Those for whom $(1-\Sigma t)=0$ have necessarily been omitted. In parentheses, we show the proportion of the sample for which no change in the appropriate marginal rate occurs.

²¹ Assuming no earned income from the wife.

Table 1.9

**Elements of the Elasticity of the
Virtual Wage With Respect to the Gross Wage** ²²

Wife's Budget Constraint²³

$\Delta w/w$	Income Tax	NIC	Income Support	Housing Benefits	All taxes and Benefits
0.01	-0.115	-0.045	0	0.001	-0.151
	(99.6)	(99.6)	(100)	(100)	(99.0)
0.05	-0.079	-0.031	0.036	0.012	-0.062
	(98.2)	(98.1)	(99.9)	(99.5)	(95.7)
0.10	-0.061	-0.022	0.018	0.002	-0.063
	(96.7)	(96.8)	(99.9)	(99.1)	(93.0)
0.25	-0.067	-0.012	0	0.015	-0.064
	(92.0)	(94.3)	(99.9)	(98.1)	(86.9)

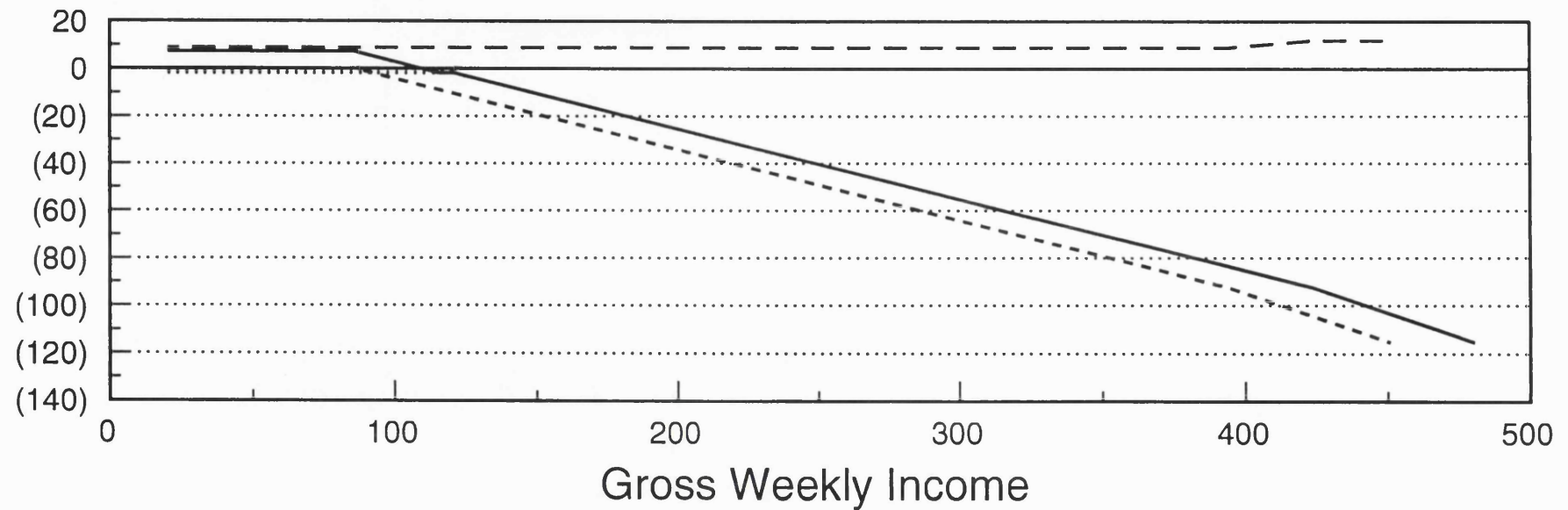
²² Table shows $[\epsilon_{(1-t),w} (1-t)/(1-\Sigma t)]$. Those for whom $(1-\Sigma t)=0$ have necessarily been omitted. In parentheses, we show the proportion of the sample for which no change in the appropriate marginal rate occurs.

²³ Taking the labour income of the spouse as given.

Figure 1.1

Elements of the Income Tax Schedule
Typical Family of Four, Owner-occupiers

Net Income Effect



Net Income Tax
Schedule

Basic Income Tax
Schedule

Composite Rate

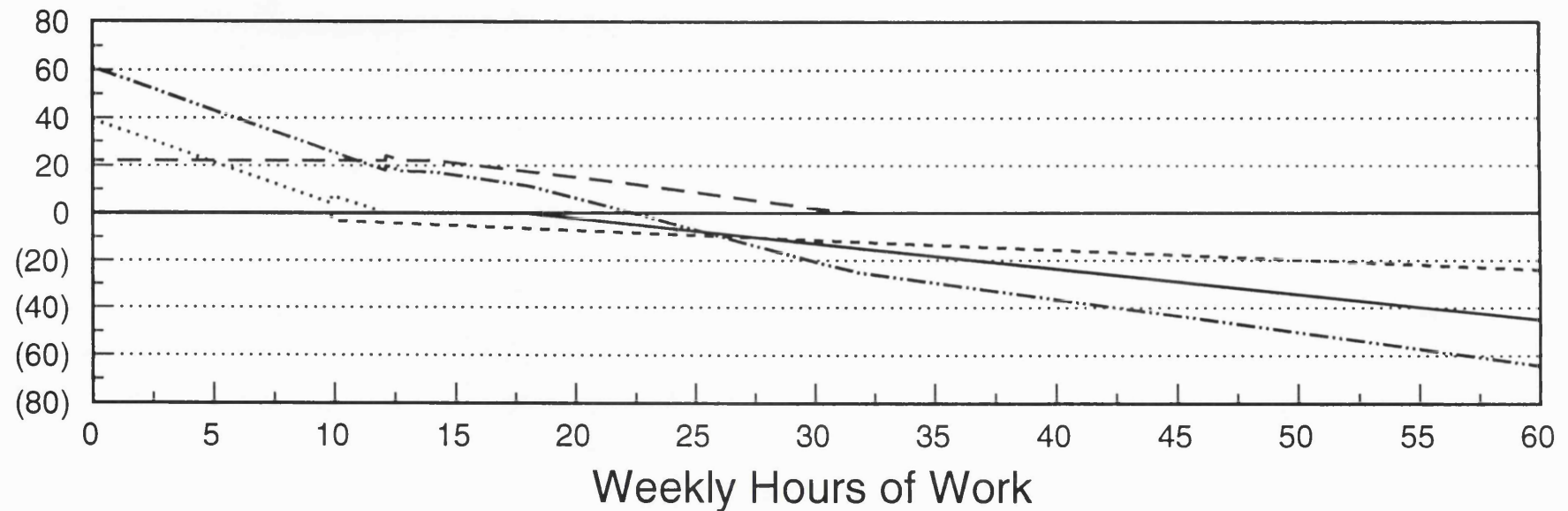
MITR

Assuming no earned income of the spouse

Figure 1.2

Elements of the Weekly Global Budget Constraint
Typical Family of Four, Renting Accommodation

Net Income Effect



Income Tax

NIC Contributions

Income Support
and Supplement

Housing Benefits

Net Effect

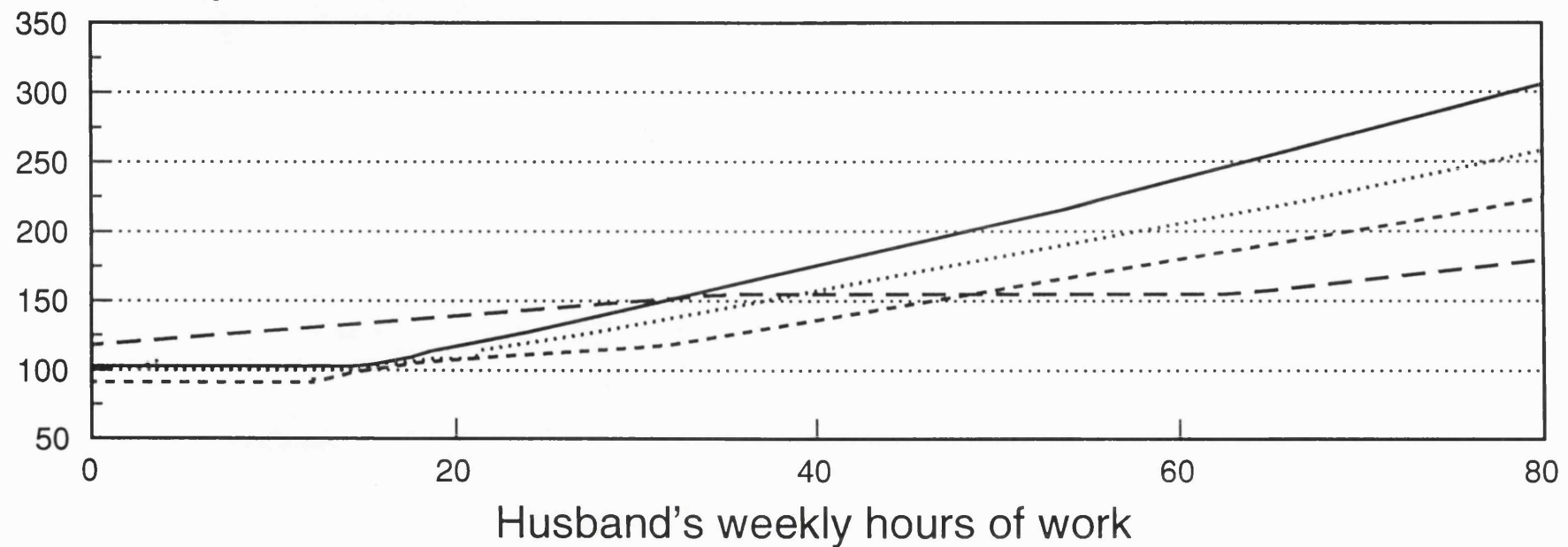
Assuming no earned income of the spouse

Figure 1.3

Global Budget Constraint

Various Typical Income Units

Net weekly income



Family of 4,
owner-occupiers

Family of 4,
renting

Family of 4,
unemployed head

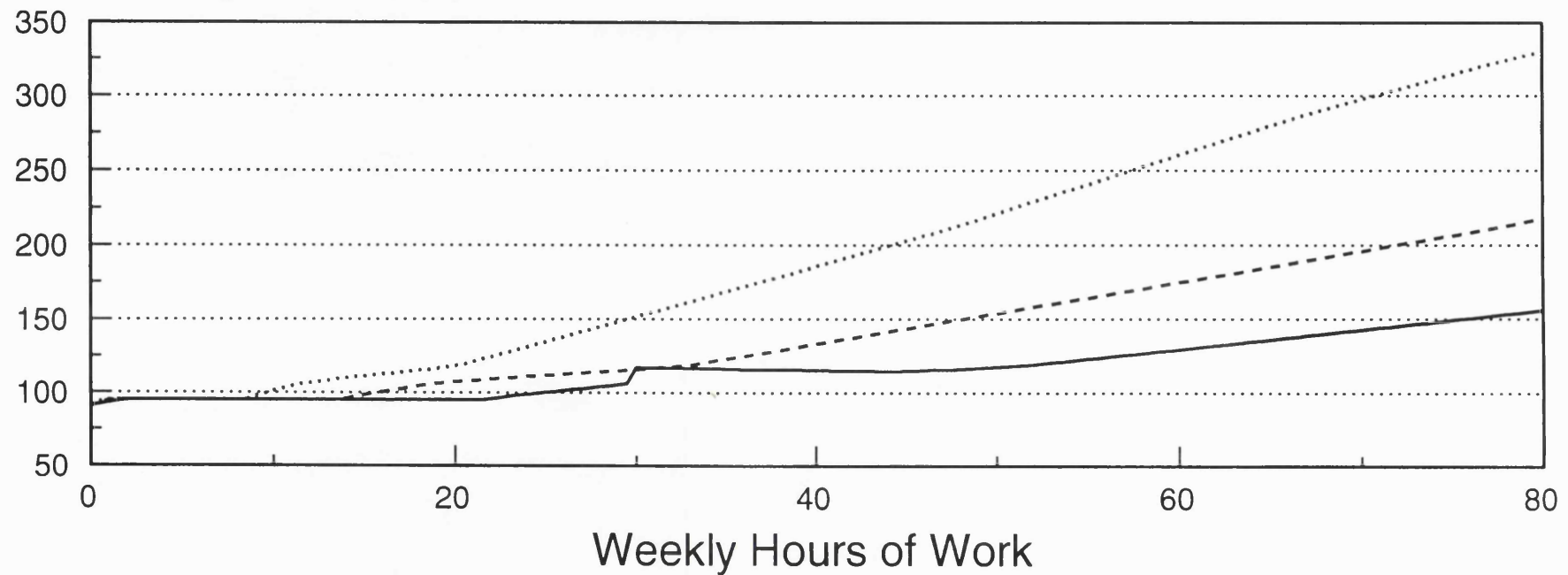
Pensionable couple
owner-occupiers

Assuming no earned income of the spouse

Figure 1.4

Budget Constraint and Wage Variability

Net Weekly Income (£)



First quintile Third quintile Fifth quintile
w=£2.19 w=£3.44 w=£5.6
 _____ - - - - -

Built from the mean wage (w) of 3 wage quintiles of families of 4, renting; characteristics other than w are from averages of the whole set

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Figure 1.5

Budget Constraints of an Average Family of Tenants

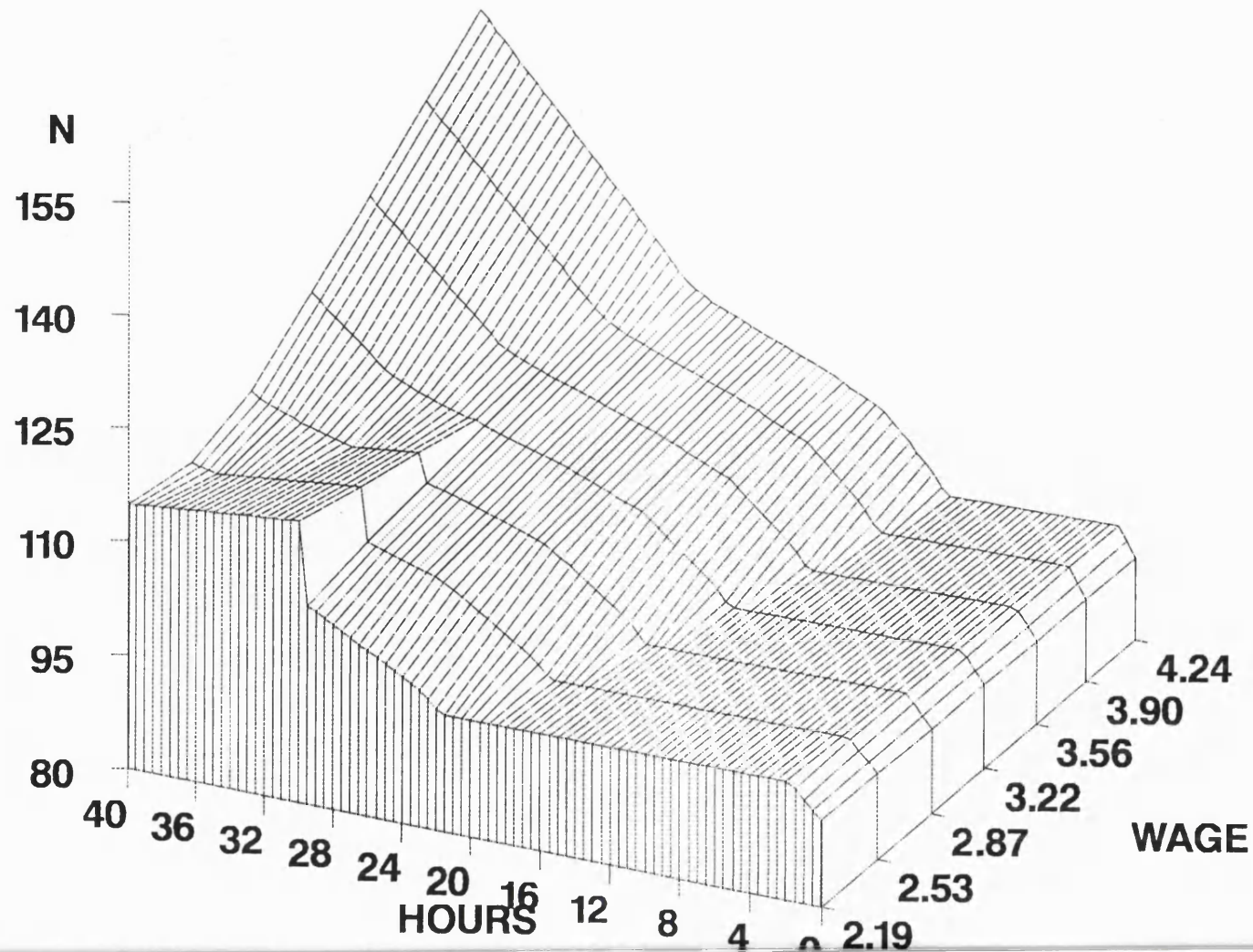
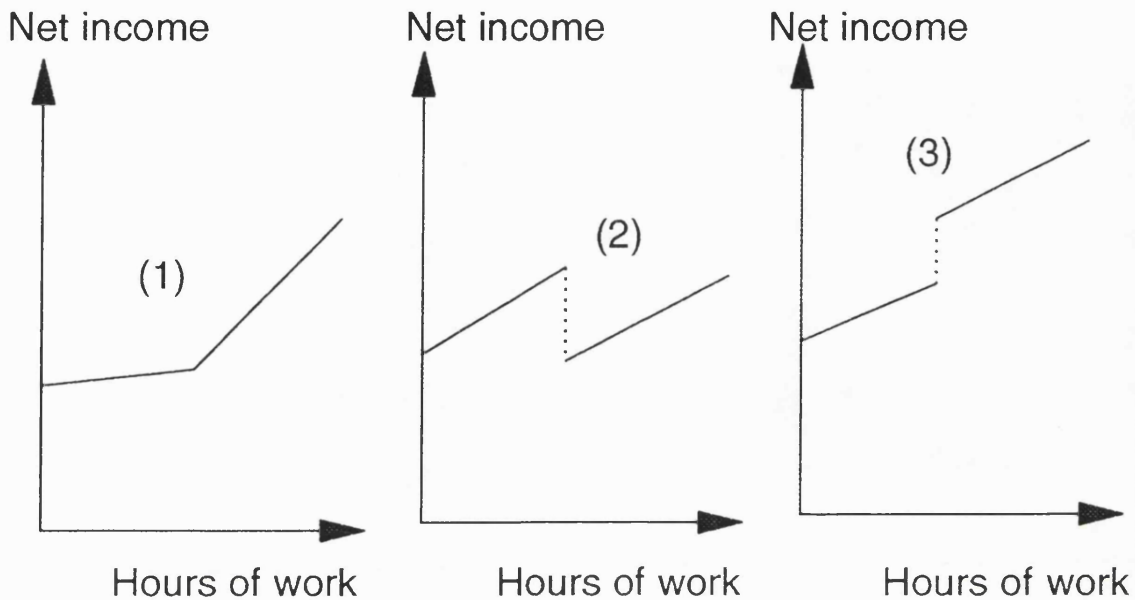


Figure 1.6a

Budget Constraint Nonconvexities



(1) Increase
in net wage

(2) Discrete fall
in net income

(3) Discrete increase
in net income

Figure 1.6b

Analytical Budget Constraint

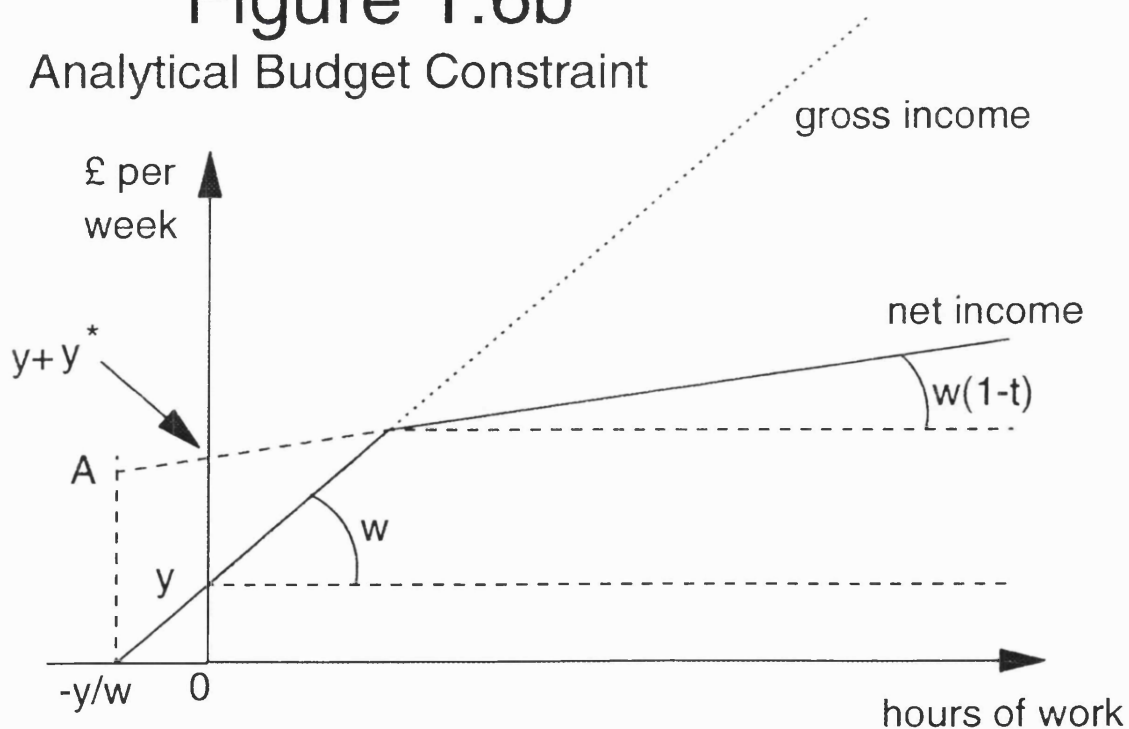


Figure 1.7

Notches

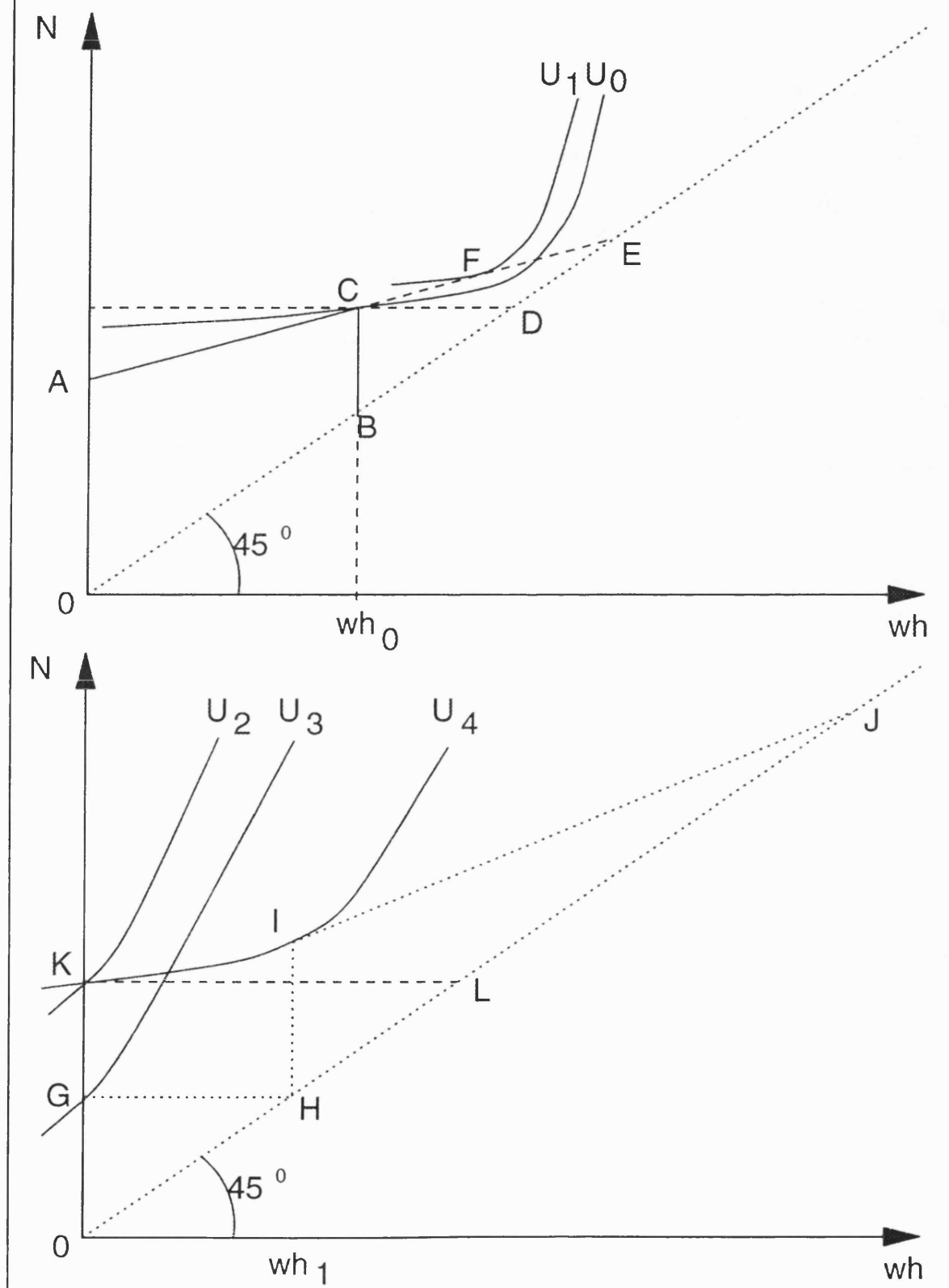
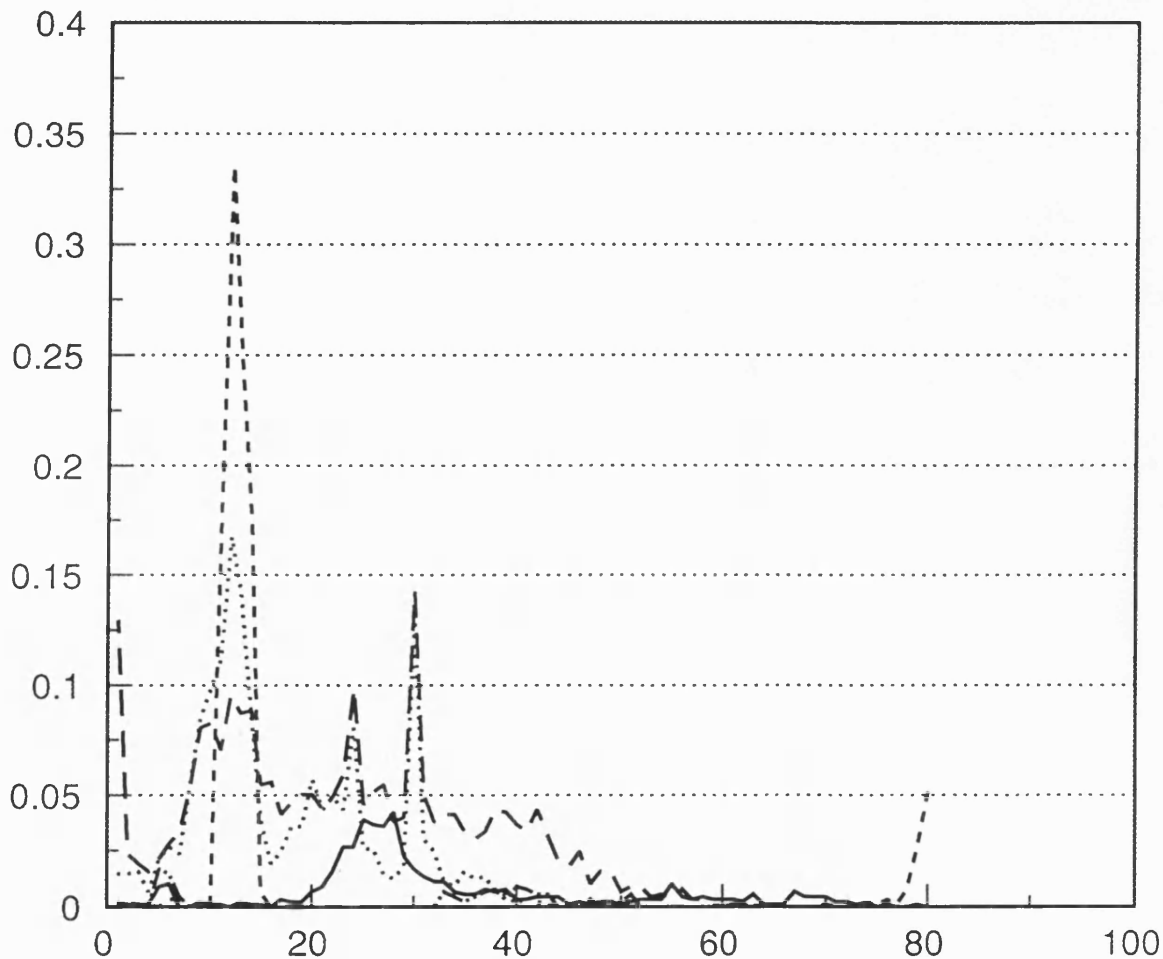


Figure 1.8

Sources of Budget Constraint Nonconvexities Heads of Income Units

Average Number of Nonconvexities in Hours Range



Income Tax

NIC

Income Support
and Supplement

Housing Benefits

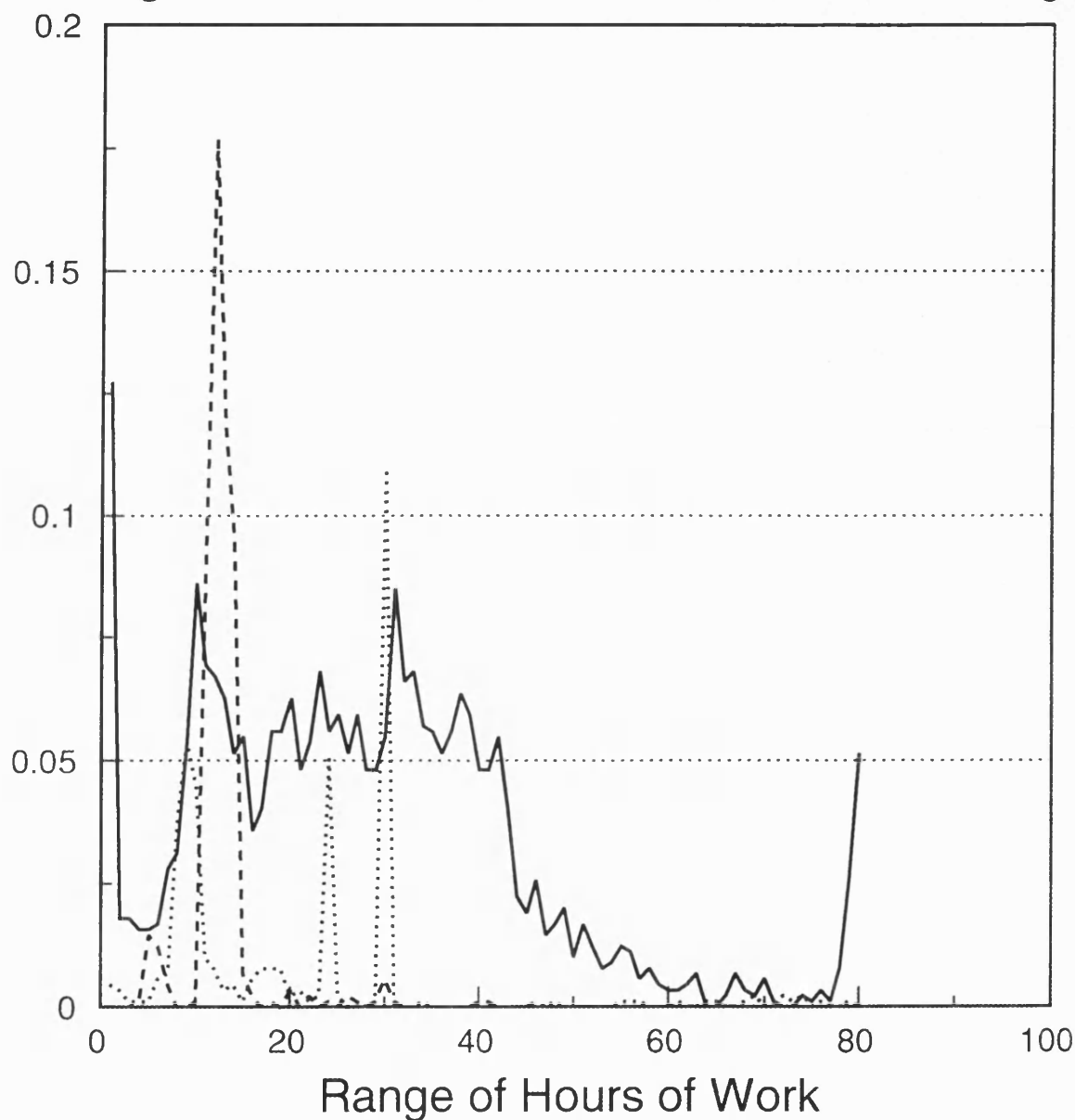
Unemployment Benefits
and Basic Pension

Assuming no labour income from the spouse
Data from the middle wage quintile

Figure 1.9

Types of Budget Constraint Nonconvexities Heads of Income Units

Average Number of Nonconvexities in Hours Range



Increase in the virtual wage

Discrete fall in
net income

Discrete increase in
net income

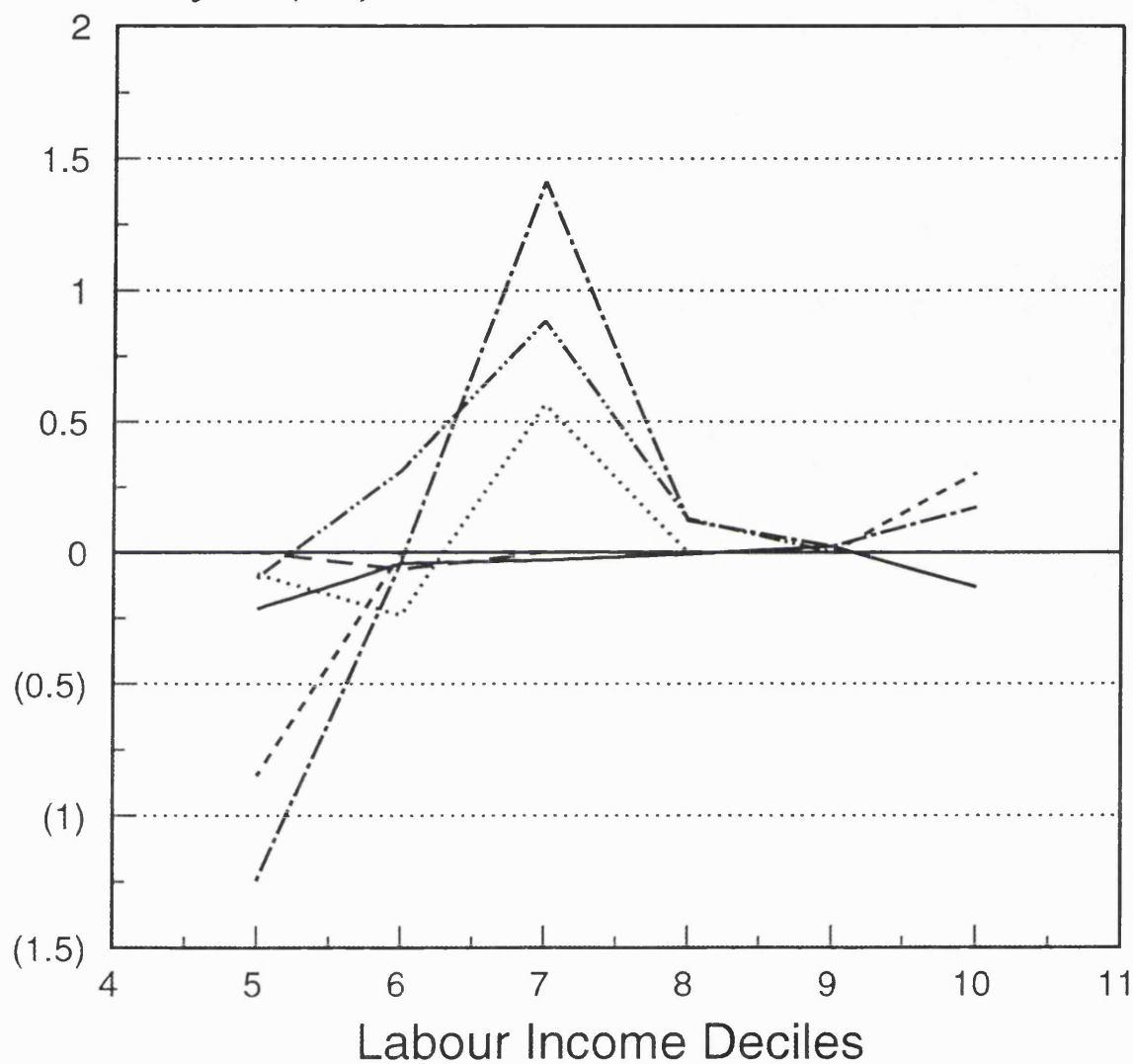
Assuming no labour income from the spouse
Using data from the middle wage quintile

Figure 1.10

Elements of the Virtual Wage Elasticity

Head of Income Unit

Elasticity of (1-t)



Income Tax

NIC

FIS and SUP

UNB and BP

Housing Benefits

Net Tax and
Transfer System

$dw/w=0.05$

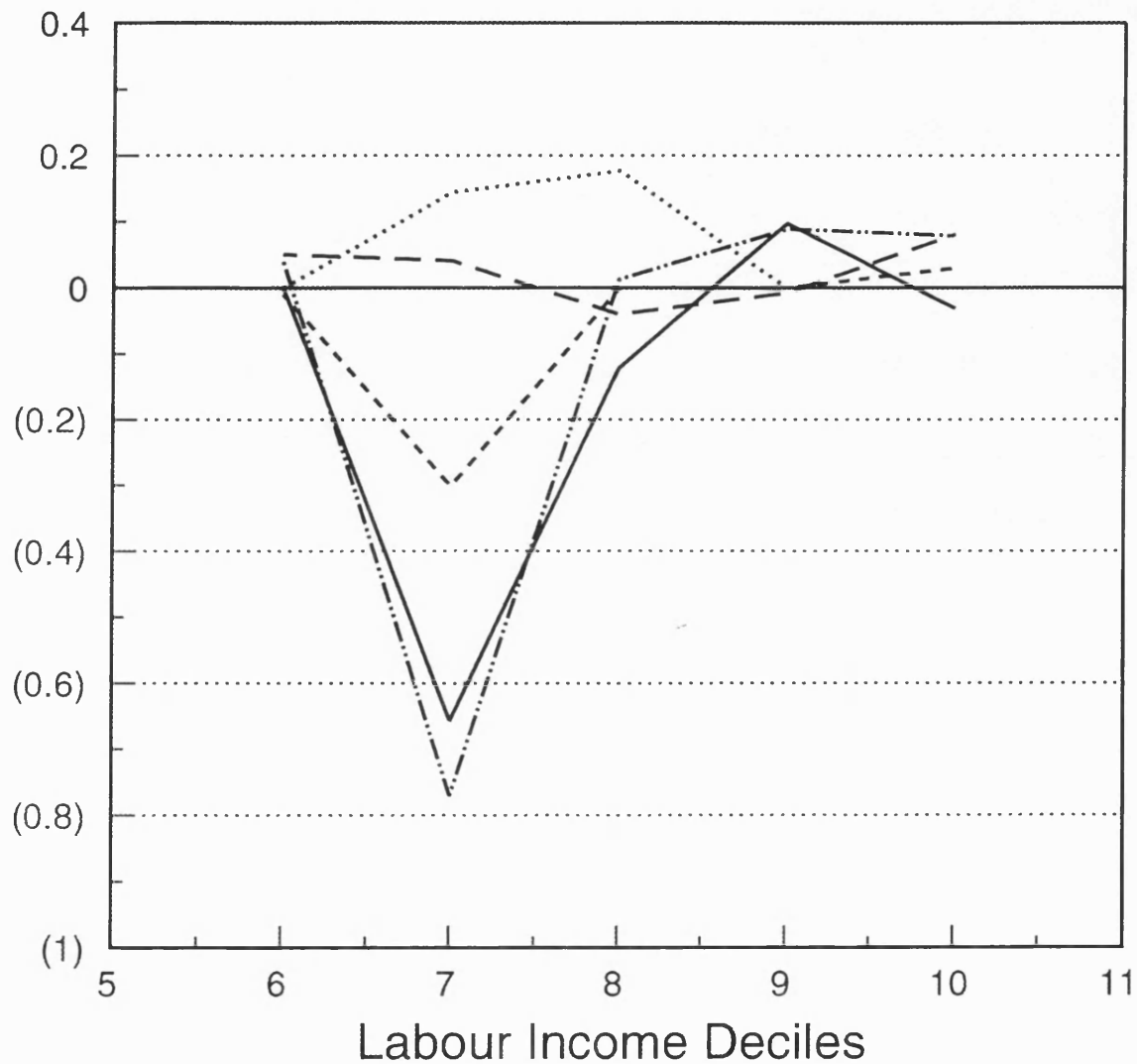
43% of the heads had no labour income

Figure 1.11

Elements of the Virtual Wage Elasticity

Wives

Elasticity of $(1-t)$



Income Tax

NIC

FIS and SUP

Housing Benefits

Net Tax and Transfer System

$dw/w=0.05$

52% of the wives had no labour income

Chapter II: Progressivity, Redistribution and Equity, with Application to 1985 Britain

Introduction

We wish here to extend and use some of the measures proposed recently [see, for example, Kakwani (1977a, 1986), Pfähler (1987) and Lambert (1989)] to portray the progressivity and redistribution of tax and benefit systems. We have four main objectives.

Firstly, we analyse and apply indices of local progressivity. Through this, we also suggest ways in which the sole use of such indices of local progression can yield interesting insights on the impact of alternative tax and benefit systems upon the level of social welfare. Secondly, we derive a general class of horizontal inequity indices that also correct for biases to the class of indices of progressivity and redistribution proposed by Pfähler (1987). We see that in the absence of such corrective horizontal inequity indices, the above indices of progressivity and redistribution have no clear implications for the real redistributive impact of taxes and benefits. Thirdly, we emphasise the analytical separate contribution of various taxes and benefits to the overall progressivity, redistribution and equity of the complete tax and benefit system. To do this, we decompose our indices of progressivity, redistribution and equity into functions of the separate impact of individual taxes and benefits. Fourthly, we illustrate the use of the developed analytical tools by applying them to the 1985 British tax and benefit system. This reveals, *inter alia*, the extent to which actual individual tax and benefit provisions are progressive, horizontally and vertically equitable, and redistributive.

The measures considered in this chapter thus fall into two categories. The first one regroups local measures of progression (*ARP*, *RP*, *LP*) for which we implicitly wish to consider representative income units whose labour income we then vary. The second category will extend the analysis to subsume the complete diversity of needs, incomes and characteristics which our sample portrays, and will thus enable us to discuss the issues of global progressivity and redistribution¹.

We proceed in two main steps. The first part of the chapter introduces measures of local progression; for this, the notation and some of the initial definitions of our analysis are inspired by Lambert (1989). We derive the contribution of individual tax and benefit components to the overall local progression of the system, applying our results to the 1985 tax and benefit regime. We then indicate how the use of local progression can provide useful benchmarks for the comparative impact of taxation and transfers on social welfare. We develop in the second part a general class of indices of horizontal inequity that is a natural complement to general classes of indices of progressivity and vertical equity. This is followed by a detailed application of our general results to an important member of the classes of indices discussed in the preceding section. Having highlighted some of the features of previous enquiries into the progressive and redistributive impact of taxes and benefits and noted the methodological

¹ The local measures of progression *ARP*, *RP* and *LP* are labelled measures of "structural progression" by Musgrave and Thin (1948), whereas our global measures would fit under their umbrella of "effective progression", where effective (or global) progression differs from "measures of structural progression" in that it is useful primarily for measuring the degree of progression of the rate structure as a whole, whereas structural progression is applicable to specific points of income on income ranges only" [Musgrave and Thin (1948), p.511].

limitations of applied and comparative analyses, we finally indicate how separate elements of the 1985 British tax and benefit system contribute empirically to the global progressivity, vertical and horizontal equity, and redistribution of the state's intervention.

A- Measures of Local Progression

1- Definitions

We defined in Chapter I a progressive tax or benefit as one for which the average rate of tax (or minus the average rate of benefit) increased with gross income. Our first indicator, the average rate progression (ARP), is the derivative of the average rate of tax with respect to gross income². Using the notation of Chapter I and taking one linear segment, ARP is defined by:

$$ARP \equiv \frac{d \left[\frac{T(N(wh))}{X(wh)} \right]}{d(wh)} = \frac{ty + y^*}{X^2} = \frac{1}{X}(t-a) \quad (1)$$

The average tax rate $a(wh)$ will be monotonically increasing with wh only if $ty > -y^*$ or $t > a$ holds; if that condition holds for each of the J_i vectors of programme (benefit or tax) i (viz, over the whole domain of incomes), programme i may be said to be progressive at all income levels, given the socio-economic characteristics of the particular income unit. In other words, in a society with income units identical in all things except in the income they derive from supplying labour, programme i will be progressive over all such distributions of labour income if

² ARP is termed "effective rate progression" by Slitor (1948).

and only if $y^* > -t y$ or $t > a$ for all linear segments $j, j=1, J_i$.

A second measure, residual progression (RP), yields the elasticity of net income to gross income:

$$RP \equiv \frac{d N(wh)}{d X(wh)} \cdot \frac{X(wh)}{N(wh)} = \frac{(1-t)(wh + y)}{(1-t)wh + y + y^*} = \frac{1-t}{1-a} \quad (2)$$

For net income N to progress less quickly than gross income X , RP needs to be below 1; for this to hold, or for $d(RP)/d(wh) > 0$ over the linear segment, we require that

$$(1-t)y < y + y^* \quad (3)$$

which is equivalent to the ARP condition for progressivity defined above. Given a net wage $w(1-t)$, this condition states that for the relevant linear segment to exhibit progressivity, the virtual income $(1-t)y$ of a proportional regime must be lower than the full virtual income of the non-proportional system, $y+y^*$. Alternatively, we can illustrate this condition using Figure 1.6b. We extend the gross income line to the horizontal axis and meet the point $-y/w$; the intercept of the net income line above point $-y/w$ is A , with a value of $y+y^*-w(y/w)(1-t) = y^*+ty$. Condition (3), which can be rewritten as $y^*+ty > 0$, then simply requires that, for a linear segment to exhibit progressivity, the intercept A must lie above the point $-y/w$ on the horizontal axis.

We also make use of the elasticity of taxes (or benefits) with respect to gross income, which is termed liability progression (LP), and which is described as

$$LP \equiv \frac{d [T(N(wh))]}{d (X(wh))} \cdot \frac{X(wh)}{T(N(wh))} = \frac{t}{a} = \frac{t}{\frac{t wh - y^*}{t wh + t y}} \quad (4)$$

It is defined only if $a \neq 0$, where a is the average tax rate. From the definition of ARP and RP and for positive t and a , we note that the following are equivalent: $LP < (>) 1$, $t < (>) a$, $RP > (<) 1$, and $ARP < (>) 0$. We also see that $ARP = 1/X^2 [T(LP - 1)]$ and $RP = 1/(1 - a) - aLP$. These last expressions will be useful below when we wish to show the distinct impact of taxes and benefits upon the ARP and RP progressivity of the global tax and benefit system.

Empirical labour supply studies whose net wage and virtual income are constant implicitly assume that global RP and LP are equal to one for all linear segments. We also note that there is no straightforward correspondence between the concept of marginal wage elasticity introduced in the previous chapter and the RP and LP measures of progression used in this one. A system with a constant marginal tax rate (and thus a constant marginal wage elasticity) may yet be progressive if the average tax rate "a" is smaller than the marginal one, t . Conversely, a decreasing marginal wage as wh increases does not necessarily imply progression of the tax and benefit system if the marginal tax rate is then still below the average one.

2- The 1985 Tax/Benefit System

We may now illustrate the local progressivity of the 1985 tax and benefit system. We recall that wherever ARP is positive, or RP is below one, the relevant scheme is said to be *locally* progressive. These, however, are simple qualitative rules for the presence or absence of progressivity and it remains not always clear

how to expect the quantitative values of ARP, RP and LP to evolve for a tax/benefit system that satisfies the qualitative progressivity conditions. In other words, having established that a system is progressive or not, can we then establish from the numerical values of ARP, RP and LP how progressive (or regressive) it is over the whole of gross income? As can be seen from the definitions of ARP, RP and LP, all three measures summarise in various ways the relationship between the marginal (t) and the average (a) tax rates. ARP appears the less useful of the measures when one must compare progressivity at different levels of gross income since $t-a$ is divided by X , which makes ARP falls inexorably to 0 regardless of any (reasonably) evolving relationship between t and a [see for instance Slitor (1948)]. It is easier to conceive of relatively unchanging RP and LP; for an average tax of 20% and a marginal one of 40%, RP would equal 0.75 and LP, 2.0. To keep such RP and LP roughly constant over the whole range of gross income when marginal tax rates in excess of average rates inevitably raise the latter, a progressive system would need to impose continuously greater marginal tax rates. This would be particularly difficult for a constant degree of LP since required marginal tax rates would sooner or later need to surpass 100% – a relative weakness noted by Musgrave and Thin (1948). Because of this, there is also in the case of LP a natural tendency for the measure to become closer to 1 as we measure it over an increasing level of gross income. RP is less susceptible of being affected by the constraint of marginal tax rates not exceeding 100% and therefore shows the greatest potential for it not to indicate an inevitable gradual loss of structural or local progressivity as income increases.

With these remarks in mind, we may then use Table 2.1 to compare the

progressivity of personal income taxation with that of the net tax/benefit system for an "average" owner-occupier family of four, whose main characteristics are displayed on Table 1.1. The net tax/benefit effect appears to be always more progressive than the income tax one, except for gross incomes in the area of £360 or above per week. It is also clear that (judging from the variations in RP) both the personal taxation and the net tax/benefit systems lose (as somewhat expected) their progressive bite as we move up the gross income scale. Further analysis reveals that all components of the benefits modelled are progressive over the whole space of incomes but that NIC, the composite tax scheme, MITR, the deductibility of half the class 4 National Insurance contributions, and the separate taxation provision can all exhibit regressivity over at least some range of the space of gross incomes, depending on the chosen household characteristics.

To grasp the extent to which individual elements contribute to overall progressivity (or regressivity), we may split net ARP into a weighted sum of LP_i :

$$Total \ ARP = \frac{1}{X^2} \left[\sum_{i=1}^m T_i \left(\frac{dT_i}{d(wh)} \cdot \frac{X}{T_i} \right) - \sum_{i=1}^m T_i \right] = \frac{1}{X^2} \left[\sum T_i (LP_i - 1) \right] \quad (5)$$

where LP_i is the liability progression of the individual tax or benefit i . Similarly, total RP can be disaggregated into a sum of LP_i as follows:

$$Total \ RP = \frac{X}{N} - \sum_{i=1}^m \frac{d(T_i)}{d(wh)} \cdot \frac{X}{N} = \frac{1}{1-a} - \sum_{i=1}^m \frac{T_i}{N} \cdot LP_i \quad (6)$$

with $a = \sum T_i / X$.

Table 2.2 displays the contribution of various income tax provisions to

overall local income tax progressivity as defined in equations (5) and (6), and this, for the same typical owner-occupier family of four. Net RP appears as a sum of TLP_i [itself a transformation of LP_i , defined as $T_i LP_i / N$], and net ARP as a sum of $TLP1_i$ [a transformation of $(LP_i - 1)$, defined as $T_i (LP_i - 1) * 10^4 / X^2$]. TLP_i and $TLP1_i$ are thus intended to summarise the individual progressive impact of taxes and benefits upon the overall average rate progression or residual progression of the system. Thus, for $X=160$ for instance, we find that global ARP equals 13.55 and represents the sum of the elementary income tax $TLP1$ (10.07) and the $TLP1$ for the Mortgage Interest Tax Relief provision (3.48). RP can similarly be traced back to $1/(1-a)$ minus the sum of individual TLP . In the dimension of gross incomes, we can conclude that net income taxation shows a monotonic loss of progressivity as measured by RP and LP (RP increases and LP falls, towards one) except in the area of $X=£120$, where income tax is regressive. This last feature is caused by the composite rate scheme rules. In the dimension of the elements adding up to net income taxation, we may say that (for this particular income unit) net income tax progressivity is superior to that of the "elementary" income taxation (viz, reinforced by other tax provisions), except for the case of $X=£120$ already mentioned and for $X=£400$, where MITR effectively begins to allow the deduction of mortgage interest payments against income belonging to higher tax brackets.

Net tax/benefit progressivity for a family of four (tenants) is similarly shown in Table 2.3. The contribution of income tax to overall progressivity is less than half for gross incomes at or below £120 a week. NIC contributes regressively for $X=£320$ and above, and thereby attenuates the already low progressivity of personal income taxation at high income levels.

3- Residual Progression and Social Welfare

We have already discussed the difficulties involved in interpreting the size and the quantitative attributes of local progression introduced at the beginning of this chapter. We have said that quantitative uses of ARP did not appear to be potentially useful, and that we did not expect RP and especially LP to increase or even to stay relatively constant as gross incomes increased in a sensible tax and benefit system -- this limitation being due to the practical problems inherent in setting very high marginal tax rates. A related difficulty in the comparative use of local measures of progression is that a system may exhibit strong progressivity at some income levels but not at others, while the converse holds for another system to which we would like to compare the first one. One way of sidestepping these difficulties and of indeed characterising the quantitative and comparative merit and impact of variously progressive tax and benefit systems is through the adoption of an explicit social welfare function as a measuring rod. This would also bring us closer to the analytical framework of the next section, where we concern ourselves with aggregate or global measures of progressivity and redistribution.

We thus pause for a moment to illustrate the quantitative redistributive gains that might be achieved when switching from a proportional tax system $N=(1-\lambda)X$, with average and marginal tax rates equal to λ , to a system, $N=AX^\theta$, with a constant residual progression $RP=\theta$ throughout the gross income distribution³. To do this we adopt the following definition for $M(\sigma)$, the mean of

³ X and N are not "equivalent" or "equivalised" incomes, but simply the unscaled original and net incomes of the various income units. For an early use of this function and a critical and encompassing discussion of the principles and application of

order σ :

$$M(\sigma) = \begin{cases} \frac{1}{\sigma} \int_{XB}^{XU} X^\sigma f(X) dX, & \text{if } \sigma \neq 0 \\ \int_{XB}^{XU} \ln(X) f(X) dX, & \text{if } \sigma = 0 \end{cases} \quad (7)$$

where XB and XU are respectively the lower and upper limits of the gross income distribution of X , and $M(1)$ is nothing but the mean. Under a constant RP regime, the average tax rate \bar{t} will equal:

$$1 - \frac{A \theta M(\theta)}{M(1)} \quad (8)$$

We make use of the social welfare functions of the type:

$$SWF = M(1 - \epsilon) \quad (9)$$

Such social welfare functions imply a marginal social utility of individual income equal to $X^{-\epsilon}$. A rich person with twice the income of a poor one would thus exhibit a social marginal utility of his income 0.6 time that of the poor for $\epsilon=0.75$, and 0.84 times for $\epsilon=0.25$. According to this criterion, for $\epsilon=0.75$, it would thus be socially acceptable (at the margin) in the redistributive process to waste up to 40% of the resources reallocated from the rich to the person with half his

progressive taxation, see Vickrey (1947). Jakobsson (1976) implicitly encourages the use of such a function by indicating that "as soon as the context chosen is income redistribution judged by the Lorenz criterion, there is just one logical measure of progression" (p.161), that is, RP.

income (16% when $\epsilon=0.25$): at the margin, the cost in "wasted" resources would then just equal the welfare gain in redistributing income to the less well off.

Substituting the definition of the above tax systems into that of social welfare, we note that social welfare equals:

$$SWF = \begin{cases} (1-\lambda)^{1-\epsilon} M(1-\epsilon), & \text{proportional system} \\ A^{1-\epsilon} \theta M((1-\epsilon)\theta), & \text{constant RP} \end{cases} \quad (10)$$

where, again, $N=(1-\lambda)X$ for proportional taxation and $N=AX^\theta$ for the constant RP system. The social welfare function of the constant RP system differs from that of the proportional system by the constant term $(1-\lambda)^{(1-\epsilon)}$ and $A^{(1-\epsilon)}\theta$, and by the fact that it deals with a mean of order $(1-\epsilon)\theta$ rather than $(1-\epsilon)$, the additional θ being generated by taxation with a constant level of residual progression.

We can then compare the social welfare attained under a purely proportional tax system (with the net average tax/benefit rate, $\lambda=7.1\%$, equal to that calculated by our computer model of the 1985 British tax and benefit system) with that of a constant RP system. It is possible to identify the value of the constant A in $N=AX^\theta$ for which such social welfare will be equal, given a particular pair of (θ, ϵ) :

$$A = (1-\lambda) \left[\frac{M(1-\epsilon)}{\theta M((1-\epsilon)\theta)} \right]^{1/(1-\epsilon)} \quad (11)$$

It is the case that, for equal social welfare and for all $\epsilon>0$, a lower value of θ (viz, stronger residual progressivity and redistribution) will yield a higher average tax rate \bar{t} of the constant RP system. In other words, in achieving the

same level of social welfare as that of a purely proportional system, we can augment (or lower) the progressive bite of a constant RP system to make it generate more (or less) tax revenues. The difference between the average tax rate of a purely proportional system with that of the progressive constant RP system for the same "social welfare" and for various θ will then indicate the potential for additional tax revenue which the progressive regime would allow, for a given SWF; it represents:

$$(\bar{t} - \lambda) = (1 - \lambda) \left[1 - \frac{M(\theta)}{M(1)} \theta^{-\epsilon/(1-\epsilon)} \left(\frac{M(1-\epsilon)}{M(\theta(1-\epsilon))} \right)^{1/(1-\epsilon)} \right] \quad (12)$$

It can be seen that such a "RP gain" will equal 0 in the case of $\theta=1$ (no residual progressivity) or $\epsilon=0$ (income-utilitarian social welfare function).

Figure 2.1 shows the value $\bar{t}-\lambda$ of such differences for various inequality aversion parameters ϵ and RP values θ . The computations include the gross or original income of all the units of our sample and make no allowance for differences in needs or unit size and composition. The greater the value of θ , the lesser the excess average tax revenues $\bar{t}-\lambda$ which RP can extort for the same SWF as that of a purely proportional system; equivalently, the greater the degree of progressiveness (viz, the lower θ), the greater the average tax revenue that can be allocated for the same SWF. For a value of ϵ equal to 0.75, a conversion from a proportional to a RP progressive system could lead to an additional tax collection of up to 47% of gross incomes without a fall in the level of social welfare. We note a discrete fall in the value of $\bar{t}-\lambda$ at $\theta=0$ since all are then assigned an identical net income, even those who reported no original income at all.

Wielding the above tools also allows us to compare the "revenue efficiency" of the 1985 tax and benefit system to that of a purely proportional one. More precisely, we wish to calculate the percentage gain in state revenue which a hypothetical switch from a proportional system to the 1985 system would have allowed, given the level of social welfare attained under the proportional regime. We thus wish to equate $(1-s)^{(1-\epsilon)}M^*(1-\epsilon)$, the adjusted level of the 1985 social welfare, to $(1-\lambda)^{(1-\epsilon)}M(1-\epsilon)$, the social welfare of a proportional system with tax rate equal to λ . s is the additional proportional tax necessary to equate the level of social welfare in the two regimes. M^* is the level of social welfare achieved under the 1985 tax and benefit rules. The total average tax rate under the "welfare-equivalent" 1985 system thus equals $1-(1-\lambda)*(1-s) = \lambda+s-\lambda s$. The difference between the average tax rates of the two regimes is then $\lambda+s-\lambda s-\lambda = s(1-\lambda)$. Solving for the value of s that equals the social welfare under the proportional system to that of the 1985 regime, we find:

$$s = 1 - \left[(1-\lambda) \left(\frac{M(1-\epsilon)}{M^*(1-\epsilon)} \right)^{1/(1-\epsilon)} \right] \quad (13)$$

Calculating the values of the difference $s(1-\lambda)$ in average tax rates for the 1985 system and data yields for the 1985 system and data $s(1-\lambda)=7.2\%$ for $\epsilon=0.25$, $s(1-\lambda)=17.4\%$ for $\epsilon=0.50$, and $s(1-\lambda)=35.0\%$ for $\epsilon=0.75$. Hence, the application of the non-proportional 1985 (as opposed to a proportional one, for the same social welfare) can be seen to yield additional (*yet only potential*) tax revenues ranging from 7% to 35% of gross incomes, depending on the choice of ϵ . The greater the ϵ , and therefore the more sensitive is the social welfare function to inequalities in

net incomes, the better is thus the performance of the 1985 system relative to a purely proportional one.

We have, up to now, compared the yields of a constant RP system relative to that of a proportional one, and the yield of the 1985 tax and benefit system compared to a proportional one, for a constant level of SWF in both circumstances. We can also compare the "gain" made under the 1985 system to that made under an hypothetical constant RP system, with varying levels of θ , always with reference to the level of social welfare obtained under a proportional regime. Looking at Figure 2.1, we search for the value of θ which would yield a similar tax bonus to that of the 1985 system, for a given value of ϵ . If ϵ equals 0.25, the "yield" of the 1985 regime is comparable to that of a constant RP system with $\theta=0.38$; that value drops to $\theta=0.30$, with a consequent increase in residual progressivity, if $\epsilon=0.50$. This suggests that, for a SWF more sensitive to inequalities in net incomes (greater ϵ), the lower is the equivalent value of θ (the greater is the equivalent value of constant progressivity). For relatively large values of ϵ (such as $\epsilon=0.75$ on the figure), the 1985 system dominates all forms of constant RP regimes -- except that for where $\theta=0$, for which net incomes are identical -- since such regimes fail to redistribute sufficiently large amounts to those with very low (or no) incomes. More precisely, those with no (recorded) original income in the FES do not benefit from any degree of redistribution achieved under $N=AX^\theta$ since N is always zero when $Y=0$, except when $\theta=0$ where our computer package attaches a value of 1 to X^θ for all real X . An immediate lesson from this exercise -- which could easily be extended to the consideration of single tax or benefit elements, or of various combinations of them -- is thus that trying to approximate

the degree of overall residual progression of an existing tax and benefit system is likely to depend strongly on the choice of the parameter ϵ and on the distribution of original incomes.

B- Measures of Global Progressivity and Redistribution

1- Introduction

The way in which we have defined the term "progressivity" up to now is not generally sufficient to grasp the extent to which various taxes and transfers are redistributive. Contrary to what was assumed in most of the previous analysis, income units differ by more than their labour income: their taxable and non-taxable non-labour income also vary, and so do their needs and characteristics. Income units are also distinguished in ways which the tax and benefit rules make relevant: in their ability to elude taxes or their failure to claim entitlements to admissible benefits, in their housing decisions, their consumption patterns, their employment status, their age and sex, etc... For the net tax and benefit system to be progressive for all possible distributions of incomes and characteristics, we would need ARP to be positive, RP to be below one, and LP to be above one over the domain of all such distributions, a feature which the 1985 tax and benefit system obviously does not possess⁴. To say something on the *actual* redistributive impact, we will now integrate the particular sample distributions of income unit differences. To deal with differences in needs across units and to allow for the analysis to be made over all individuals (instead of income units), we will use

⁴ Lambert (1988) analyses the conditions for which a tax or benefit system which is locally progressive over the whole range of incomes of separate groups of units also decreases income inequality of the combined distribution of such different groups.

throughout the equivalence scale implicit in the 1985 Supplementary Benefit scale⁵. This procedure is disputable but will nevertheless help to focus our attention on the points most relevant to our study.

Concentration curves can be defined as⁶

$$C_{YZ} = C(p, Y, r(Z)) \quad (14)$$

They indicate the cumulative total of the first $p\%$ of the observations of variable Y (expressed as a proportion of the overall sum of Y) when such observations are ranked in increasing order of corresponding variable $Z=Z(Y)$. Thus, for a continuous distribution of Z , with $p_0 = F(Z)$, F being the cumulative density function of Z and μ_Y the mean of Y , we find that $Y(p_0)=Z^{-1}(F^{-1}(p_0))$ and that

$$C(p, Y, r(Z)) = \frac{1}{\mu_Y} \cdot \int_0^p Y(p_0) dp_0 \quad (15)$$

The ordinary Lorenz curve, $L_Y(p)$, is a special case of (15) occurring when $r(Z) \equiv r(Y)$, i.e., $L_Y(p) = C(p, Y, r(Y))$. In particular, the Lorenz curve for a continuous distribution of gross incomes X may be usefully seen as

⁵ Excluding from such a scale the element of supplementary benefit springing from the payment of mortgage interest by owner-occupiers. On the use and derivation of equivalence scales, see -- for a survey -- Buhman et al. (1988) and Coulter et al. (1992).

⁶ See Mahalanobis (1960) and, more recently, Kakwani (1977a,b). Here we adopt part of the notation of Atkinson (1979), p.9.

$$L_X(p) = \frac{1}{\mu_X} \cdot \int_0^p X(p_0) dp_0 \quad (16)$$

The Lorenz curve $L_X(p)$ thus shows the percentage of total income X which the poorest $p\%$ of the population receives. Hence, "the distance $L_{(X-T)}(p) - L_X(p)$ is that fraction of total post-tax income shifted from high incomes [the top $100(1-p)$ percent] to low incomes (the bottom $100p$ percent) by the presence of progression in the tax" [Lambert (1989), p.179].

To each concentration curve $C_{Y,Z}$ we may also assign an index $I_{Y,Z}$ of inequality in the distribution of Y , defined as

$$I_{Y,Z} = 2 \int_0^1 [(p - C(p, Y, r(Z)))] dp = 1 - 2 \int_0^1 C_{Y,Z} dp \quad (17)$$

When $r(Y) \equiv r(Z)$, $I_{Y,Z}$ simply becomes the Gini coefficient:

$$G_Y \equiv I_{Y,Y}$$

The Kakwani (1977a) index of progressivity is then defined by

$$\Pi^K = 2 \int_0^1 (L_X - C_{T,X}) dp = I_{T,X} - G_X \quad (18)$$

It indicates the size of twice the area between the Lorenz curve for X and the concentration curve for T using $r(X)$. We will see later that it enters nicely into the decomposition of an overall index of redistribution, the Reynolds-Smolensky (1977) index, itself known as

$$\Pi^{RS} = 2 \int_0^1 (C_{X-T,X} - L_X) dp = G_X - I_{X-T,X} \quad (19)$$

and which is a measure of the redistribution of resources from X to X-T. Π^{RS} shows the value of twice the area between the Lorenz curve for X and the concentration curve for X-T. "In the absence of reranking, this is the reduction in the Gini coefficient achieved by the tax – a measure of redistributive effect" [Lambert (1989), p.180].

2- A General Class of Measures and Indices of Horizontal Inequity

Kakwani(1986,1987) and Pfähler(1987) have shown that Π^K and Π^{RS} are particular members of a general class of indices of progression and redistribution. Using Pfähler's framework but incorporating our notation, this "general class of scale invariant aggregate measures of the after-tax redistribution effect (AIR) which satisfy the Pigou-Dalton principle of transfers" [Pfähler(1987), p.12] is obtained from ⁷

$$AIR = \frac{1}{\mu_N} \int_0^1 [N(F_X^{-1}(p)) - (1-g)F_X^{-1}(p)] W(p) dp \quad (20)$$

where $F_Z(Z)$ is the cumulative distribution function of Z and $Z=F_Z^{-1}(p)$ is the

⁷ Where g is here the average "equivalent tax" rate, that is, the average rate of equivalent tax applied to the equivalent income X.

inverse of such a distribution, and where $W(p)$ is a monotonic weight depending on p . AIR is thus a weighted sum of the differences between net income obtained from the actual tax and transfer system and net income that would follow from a purely proportional system. For weights monotonically decreasing, the more redistributive is the tax system, the greater will AIR be. The converse holds true for monotonically increasing weights. Π^{RS} is a special case of AIR and is obtained when $W(p)=-2p$.⁸

The above AIR definition can be rewritten as

$$AIR = \frac{1}{\mu_N} \int_0^1 F_N^{-1}(p) W[F_X(N^{-1}(F_N^{-1}(p)))] dp - \frac{1-g}{\mu_N} \int_0^1 F_X^{-1}(p) W(p) dp \quad (21)$$

with $N^{-1}(N)=X$. The weight attached to $N=F_N^{-1}(p)$ is thus directly dependent upon the pre-tax income distribution F_X . Unless [as Pfähler(1987) assumes throughout his analysis] $F_X(X) \equiv F_N(N(X))$, viz, the ranking of pre- and post-tax incomes are identical, it can be coherently argued that, for all members which the class AIR purports to encompass and which obey the axiom of anonymity in the treatment of post-tax incomes, it is not appropriate to weight N with weights that depend on the pre-tax cumulative distribution of X .

This is nevertheless what happens when the AIR class of measures is used in the presence of reranking. AIR will therefore be an inexact index of redistribution for all those measures for which it would be more appropriate to use $W(p)$ instead of $W(F_X(N^{-1}(F_N^{-1}(p))))$ in (21). We can shed further light on the

⁸ Similarly, the often used Hainsworth-Suits index (see, e.g., Hainsworth(1964)) follows from $W(p)=-2L_X$.

size of the bias introduced. The true redistributive index corresponding to the class defined by AIR may be given by ⁹

$$AIR^* = \frac{1}{\mu_N} \int_0^1 F_N^{-1}(p) W(p) dp - \frac{1-g}{\mu_N} \int_0^1 F_X^{-1}(p) W(p) dp \quad (22)$$

This form of AIR^* suggests that it might be helpful to think of the tax system as operating a movement from F_X to F_N of the cumulative distribution rather than a shift of X to N of the income distribution. Thinking in terms of F_X to F_N instead of X to N leads one to move from a dual definition of AIR^* (one based on the integration over a probability distribution p) to a primal one (one based on the integration over a range of incomes Y). The "primal" analogue of the definitions of AIR is

$$AIR = \frac{1}{\mu_N} \int_0^\infty Y W(F_X(N^{-1}(Y))) f_N(Y) dY - \frac{1-g}{\mu_N} \int_0^\infty Y W(F_X(Y)) f_X(Y) dY \quad (23)$$

Similarly,

$$AIR^* = \frac{1}{\mu_N} \int_0^\infty Y W(F_N(Y)) f_N(Y) dY - \frac{1-g}{\mu_N} \int_0^\infty Y W(F_X(Y)) f_X(Y) dY \quad (24)$$

A comparison of the primal definitions above thus confirms that AIR^* is a more natural index of the redistributive shift from X to N ; this is because in AIR the

⁹ Equation (22) simplifies to (21) when $F_N(N(X)) \equiv F_X(X)$, and thus when $r(X) \equiv r(N)$.

redistributive and implicit welfare weight granted to the post-tax income distributive $f_N(Y)$ depends on the pre-tax distribution F_X . In fact, this feature makes it possible that a tax and benefit system believed to be redistributive (or progressive) under the definition *AIR* may generate a more unequal distribution than the original one. A proposal that would, for instance, redistribute income (using a simple zero-sum transfer) from an initially richer individual to an initially poorer one would be diagnosed, using the *AIR* class of measures, as being redistributive; if, however, the transfer makes the initially poor individual richer after the transfer than the initially richer individual was at the beginning, the proposal would, under all reasonable measures of redistribution and income inequality, be judged as having made less equally distributed the distribution of post-transfer incomes.

It can then be deduced that the bias introduced by the class of measures *AIR* equals

$$AIR - AIR^* = \frac{1}{\mu_N} \int_0^1 N(p) \left[W(F_X(N^{-1}(F_N^{-1}(p)))) - W(p) \right] dp \quad (25)$$

that is, it represents a sum (weighted by N) of the reranking-induced errors in W . Equivalently, through an appropriate change of variables, the bias ($AIR - AIR^*$) can be conveniently expressed as

$$AIR - AIR^* = \frac{1}{\mu_N} \text{cov} \left[\left(N(F_X^{-1}(p)) - N(p) \right), W(p) \right] \quad (26)$$

with $N(p) = F_N^{-1}(p)$, $X(p) = F_X^{-1}(p)$. Thus, the greater (in absolute value) the correlation between the weights $W(p)$ and the reranking "adjustments" $N(F_X^{-1}(p)) - N(p)$, the

larger the difference $AIR-AIR^*$.

$(AIR-AIR^*)$ will indicate a positive (negative) bias for all monotonically decreasing (increasing) weights $W(p)$. In both cases the bias will suggest an exaggeration (by an appropriately interpreted index of redistribution AIR) of the extent to which redistribution from X to N has taken place. This bias occurs, for instance, for the above Π^{RS} and for the Hainsworth-Suits index, and more generally for all those measures for which it is inappropriate to weigh N with weights that depend on the pre-tax cumulative distribution of X . Such systematic biases being caused by a failure of the tax and transfer rules to preserve relative pre-tax positions, $(AIR-AIR^*)$ clearly suggests itself as an index of *horizontal inequity* for the general class of *vertical equity* measures encompassed by (21).¹⁰ The class of "correct" redistributive indices AIR^* can then be adequately interpreted as a sum of "vertical equity" and corresponding "horizontal inequity" measures: $AIR^* = AIR + (AIR^* - AIR)$.

Horizontal inequity as defined and reranking of income units clearly matter as an index of the departure of AIR from a more accurate depiction of the redistributive effect of taxes and benefits. They also will matter for other reasons. Typical income-utilitarian and individualistic social welfare orderings are typically functions of the distribution of final incomes and do not depend on the distribution of "pre-tax" or original incomes. This may be a serious omission since processes and means, rather than just the ultimate outcome, will generally reveal

¹⁰ For a discussion of a wider concept of horizontal inequity, see, for instance, Jenkins(1988).

the desirability of proposed reforms¹¹. Some principles, such as equal taxation and equal state support to those units "identical in all relevant respects", should not be broken unduly. Similarly, in presenting the results of simulated reforms, significant emphasis is usually put on the distribution of absolute and relative "gainers" and "losers". Political or social processes may, in fact, prevent the occurrence of reforms which cause (or, equivalently, favour the emergence of reforms which eliminate) sizeable turbulences in the absolute and relative living standards of individuals or groups of individuals. A clear example of such "turbulence" will be analysed in subsequent chapters -- the case of incomplete take-up of state benefits, by which some in a group of otherwise similar units are deterred from claiming or are simply not awarded a state benefit. Reranking of individuals and horizontal inequity can also additionally reveal the extent of the state's control of and interference in the ways of the free market and in the original distribution of income -- interference to which many [e.g., Nozick (1974)] would strongly object.

It should be noted, however, that AIR^* as a measure of redistribution is neutral with respect to pure swaps of incomes across the population. Hence, if a transformation X to N were described by the following permutation of incomes between individual i and j ,

¹¹ King (1983), for instance, introduces a measure of horizontal inequity derived from the use of social welfare functions in which the reranking of units features explicitly.

	X	N
<i>i</i>	10	15
<i>j</i>	15	10

AIR would show a redistributive gain since the grant of £5 to individual *i* would register a "more significant" impact than the reduction of £5 in individual *j*'s income. *AIR*^{*} would nevertheless record no redistributive impact, in such a way that the index of the horizontal inequity of changing the income position of *i* versus *j* would just equal the redistributive impact recorded by *AIR*. It can, of course, be argued that distributions X and N are not equivalent in social welfare, due to the presence of reranking; when this is so, the sole use of *AIR*^{*} as an indicator of welfare would not be appropriate since it would then generally exaggerate the desirability and the effectiveness of the government's redistributive plans. In other words, some redistributive programmes might be very effective in levelling the distribution of income but, by causing substantial disturbances in the reranking and the relative positions of units, may not prove socially acceptable. A possibly appropriate index of the desirability of such redistribution, might then be:

$$AIR^* + (1-\rho) \cdot (AIR^* - AIR)$$

where the first term incorporates the net redistributive gains of proposed or existing tax and benefit rules, and where $(1-\rho) \cdot (AIR^* - AIR)$ carries their cost in horizontal equity. ρ , which lies between 0 and 1, indicates the weight of redistribution vertical equity (redistribution) relative to that of horizontal equity

in assessing the desirability of various state programmes. The greater ρ , the more weight to pure redistributional aims and the less concern about the preservation of relative income positions. For ρ close to 0, no tax and benefit system that changes the ranking of units would be acceptable.

3- Progression, Redistribution and Horizontal Inequity for the Reynolds-Smolensky Index

We illustrate the analysis of the previous section by developing further the particular case of AIR which leads to the indices Π^K and Π^{RS} described above.¹² This will then help "explain why the tax system is less progressive than it appears" [Atkinson(1979), p.18]. The index of redistribution Π^{RS} of equation (19) can be further decomposed into

$$\Pi^{RS} = G_X - I_{X-T,X} = (G_X - G_{X-T}) + (G_{X-T} - I_{X-T,X}) \quad (27)$$

The first parentheses indicate the net redistributive effect from X to X-T as measured by the change in the Gini coefficient (vertical equity) and the terms in the second set of parentheses display the extent to which the Reynolds-Smolensky measure of redistribution is biased upwards by the reranking of households between X and X-T (horizontal inequity). The net effect of the terms in this second set of parentheses is always positive [since $W_{RS}(p)=-2p$ is monotonically decreasing].

We may generalise all of the above indices of redistribution and

¹² It should be noted that Π^K and Π^{RS} have all the inconvenience and some of the advantages of the widely used Gini index of inequality: among the latter, Pfähler [(1987),p.18] points out that they share "their well-understood and intensively investigated descriptive and implicit normative properties."

progression to the case in which m taxes or benefits T_i (with global average tax rate g_i) enter the final distribution of net income N . Define N for each income unit as

$$N = X - \sum_{i=1}^m T_i \quad (28)$$

It follows that

$$\mu_N = \mu_X (1 - \sum_{i=1}^m g_i) \quad (29)$$

where μ_Y is the mean of the distribution of Y . The concentration curves are defined as before, viz,

$$C_{T_i, X} = C(p, T_i, r(X)) \quad (30)$$

with

$$C(p, T_i, r(X)) = \frac{1}{g_i \mu_X} \cdot \int_0^p T_i(p_0) dp_0 \quad (31)$$

Then, using the definition of net income N and equations (29) and (30), we see that

$$C_{N, X} = \frac{1}{\mu_X (1 - \sum_{i=1}^m g_i)} \int_0^p (X - \sum_{i=1}^m T_i(p_0)) dp_0 \quad (32)$$

is the concentration curve for N with $r(X)$, and thus that

$$C_{N,X} = \frac{1}{1 - \sum_{i=1}^m g_i} \cdot \left[L_X - \sum_{i=1}^m g_i C_{T_i X} \right] \quad (33)$$

We recall that $C_{N,X}$ captures the accurate picture of the after tax (and benefit) distribution only if rankings $r(X)$ and $r(N)$ are identical, which is implausible given our discussion of the 1985 tax and benefit system. The correct redistributive picture, yielded by the switch of the pre-tax Lorenz curve to the post-tax one, is given by:

$$L_N - L_X = (L_N - C_{N,X}) + (C_{N,X} - L_X) \quad (34)$$

We can analyse total redistribution as given by the above equation by decomposing the terms in the two sets of parentheses into "reranking" and "departure from proportionality" effects of the set of taxes T_i . Further decomposition yields for the "departure from proportionality" terms:

$$C_{N,X} - L_X = \frac{1}{1 - \sum_{i=1}^m g_i} \cdot \sum_{i=1}^m g_i (C_{T_i X} - L_X) \quad (35)$$

and for the "reranking" terms:

$$\begin{aligned}
L_N - C_{N,X} &= (C_{N,X-T_1} - C_{N,X}) + (C_{N,X-T_1-T_2} - C_{N,X-T_1}) + \dots + (C_{N,X-\sum_1^m T_i} - C_{N,X-\sum_1^{m-1} T_i}) \\
&= \sum_{k=1}^m \left[C_{N,X-\sum_{i=0}^k T_i} - C_{N,X-\sum_{i=0}^{k-1} T_i} \right]
\end{aligned} \tag{36}$$

where $T_0 = 0$. L_N will always lie strictly below $C_{N,X}$ whenever principles of relative position maintenance are violated. Because of this, welfare (or equality) dominance results based on the comparison of $C_{N,X}$ with L_X instead of L_N and L_X will only yield sufficient conditions for L_N to be dominated by L_X and necessary conditions for L_N to dominate L_X .

We may transform the above results on the disaggregation of concentration and Lorenz curves into corresponding inequality indices of the type described earlier. Proceeding to the integration of (34) from 0 to 1 and rearranging, we find that the change in the Gini coefficient from distribution X to distribution N is equal to

$$\begin{aligned}
G_X - G_N &= (G_X - I_{N,X}) + (I_{N,X} - G_N) \\
&= \frac{1}{(1 - \sum g_i)} \left[\sum_{i=1}^m g_i (I_{T_i X} - G_X) \right] - \left[\sum_{k=1}^m (I_{N,X-\sum_{i=0}^k T_i} - I_{N,X-\sum_{i=0}^{k-1} T_i}) \right]
\end{aligned} \tag{37}$$

Since the Kakwani indices for T_i are

$$\Pi_i^K = I_{T_i X} - G_X \tag{38}$$

we see that the terms within the first set of square brackets form a sum (weighted by $g_i/(1-\sum g_i)$) of the m indices of departures from proportionality. The extent to

which programme i exerts redistributive power (as measured by $G_X - G_N$) is thus related to its intrinsic progressivity bite (Π^K) and to its importance g_i relative to net income. As put by Kakwani (1977a), this can help "distinguish between the effects of changes in average tax rates and in progressivity" (p.71) on the redistribution of income. The terms in the second set of brackets sum to one element of the class of horizontal inequity measures (AIR-AIR'). They indicate the extent to which separate taxes and benefits may be individually responsible for the reranking of family units and individuals and thus exhibit their bearing on the overall index of horizontal inequity.

4- Redistribution and the 1985 Tax/Benefit System

(a) Methodological Issues and Inter-Study Comparisons

Equation (37) above provides a useful basis on which to discuss the global redistributive impact of the various programmes discussed in Chapters I and II. Applying the methodological tools developed here is not, however, empirically straightforward. Defining a proper income base on which to assess the original position of units is both quite difficult and highly subjective. In particular, a crucial issue in judging horizontal inequity caused by housing subsidies and tax concessions lies in being able to define income correctly, allowing, say, for owners' imputed rents and for economic subsidies to various tenants¹³. The results obtained here also depend, of course, on the nature of the equivalence scale adopted and on many of the modelling assumptions made. Furthermore, most of the benefits are categorical and were devised to treat differently people who might otherwise appear to the analyst as similar in all observed respects.

¹³ For such a tighter definition of income, see Hills (1991b).

Granting retirement pension to those who have paid National Insurance Contributions and refusing it to those who have not might, for example, provide to the cross-section analyst a misleading case of horizontal inequity; a more proper consideration of horizontal inequity would in those circumstances deal with lifetime redistribution and reranking. Even more fundamentally, there are the important questions of whether income reranking is a good index of welfare reranking¹⁴, and of how units really would behave absolutely and relatively in a world of "original" opportunities, with corresponding levels of original incomes. Since we have neglected much of all those important issues, the results presented here should be considered mostly for illustrative purposes.

There are similarly several reasons for which it is difficult to contrast our empirical results on progressivity and redistribution with those reported in previous redistributive studies. Lambert and Pfähler (1990) highlight a fundamental factor for which such intertemporal and intercommunity comparisons are difficult: The perceived progressive and redistributive nature of a tax or benefit system typically depends both on the structure of the tax or benefit scheme under study **and** on the pre-tax or pre-benefit distribution. That is, identical tax or benefit systems will be attributed a progressive and redistributive impact that will vary along with the distribution of original income across time and communities. When contrasting the effect of different tax and benefit regimes across populations, we therefore cannot straightforwardly tell

¹⁴ Jenkins (1988) attempts to circumvent the recurrent difficulty of making welfare comparisons across units that may differ substantially in characteristics other than original income by focusing on the measurement and aggregation of reranking within relatively homogeneous population subgroups.

whether observed redistributive and progressive differences are due to the nature of the regimes or to the varying nature of the pre-tax and pre-benefit distributions. Accordingly, conclusions drawn ought typically not to apply to the intrinsic progressive or redistributive features of tax and benefit regimes but to how much they appear to contribute to redistribution and progressivity on **particular** income distributions.

Apart from this fundamental caveat, there are of course a number of methodological and empirical considerations which make inter-study comparisons difficult, if not impossible. We illustrate this and previous results by a parsimonious reference to a few previous enquiries, both British and international. In contrast to our study, Dilnot, Kay and Norris (1984) included in their study of tax progressivity in Britain the incidence of most indirect taxes (VAT, excise duties, etc...) and looked at the evolution of such progressivity between 1948 and 1982. They report that tax progressivity rose between 1948 and 1953, fell in the years to 1968, and rose again between 1968 and 1982. Their findings, however, are based on the simple evaluation of the local measures of progression *LP* and *RP* at average earnings each year, and cannot therefore be representative of the global progressive and redistributive impact of the tax system (benefits are excluded) over the years. O'Higgins and Ruggles (1981) distributes all taxes and public expenditures in the UK using a 1971 FES sample by making necessary incidence assumptions. They then compare their results to those of a previous study of the US [Ruggles and O'Higgins (1981)], and conclude that "although the details of tax and expenditure programs in the U.S. and the U.K. differ considerably, their overall redistributive impact appears to be remarkably similar" (p.322). Their

study relies on descriptive comparisons of decile groups and various tabulations of impacts across large sections of the population and no attempt is made to summarise the progressive and redistributive impact of the tax and benefit system in the form of global indices.

Kakwani (1977a) contrasts tax progressivity and redistribution across four countries (Australia, Canada, the U.K. and the U.S.) and reports that among these countries, Britain had both the most progressive and the least redistributive personal income tax system. Kakwani uses highly aggregated data and does not attempt to account for the impact of varying household sizes and characteristics on the distribution and redistribution of equivalent net incomes. Only tax payers and income as defined by the tax gathering authorities are relevant to the enquiry, and the latest years for which British results are shown are 1966-7. Income tax, however, does not redistribute income towards those poor who are not tax payers but who form an important segment of the population, and non-taxable income is a significant element of original income inequality; the inclusion of either of these features would reduce Kakwani's estimated progressivity of the British tax system. Such data and methodological differences can thus well explain why his index value of income tax progressivity for Britain (0.25) is very significantly above the one we will find below (≈ 0.10). Norregaard (1990) pushes forward such inter-community comparisons by looking at the variation in the progressivity and income redistribution of direct taxes (excluding transfers and most forms of social security contributions) across a set of OECD countries. Using data provided mostly by national tax agencies and focusing on some of the "concentration" or global measures described above, he finds that UK direct taxes are more

progressive than those of Sweden as well as being almost unambiguously more progressive and redistributive than those of the US, Germany and France. As for Kakwani, however, his definition of income is very limited and the data exclude in almost all cases those units who are not tax payers.

(b) Equity and Redistribution in 1985 Britain

With the contributions and the difficulties of these earlier studies in mind, we may now present the results of our own enquiry. We grant a random initial order to those units with the same initial gross income (this applies almost exclusively to those income units with zero recorded gross income). The Gini coefficient for the distribution of original (gross) income X then equals 0.489 but is "reduced" to 0.479 when the SB equivalence scale is applied to assign resources to all individuals, adults and children alike. This feature, typical of studies of this kind, is mostly due to the positive correlation of income with the size of the income units.

Table 2.4 summarises the contribution to global redistribution of the broad groups of state intervention which our computer model encompasses. It therefore illustrates the effect of the application of principles of vertical equity upon the change in the Gini coefficient predicted by (36). Because the values of Table 2.4 are not corrected for reranking, they overestimate the extent of redistribution, just as other members of the class *AIR* also would. Π^K , the Kakwani index of progressivity and the one on which we focus here, is equal to the difference between the index of inequality in the distribution of some tax or benefit and the Gini coefficient of the distribution of gross income, i.e., $I_{T,X} - G_X$. The inequality index $I_{T,X}$ of the distribution of some progressive or redistributive tax will always

be lower than the Gini coefficient of the original income (X) distribution, but the Gini coefficient of the distribution of a progressive or redistributive benefit will exceed G_X ¹⁵. Because of this, Π^K for either a tax or a benefit will have possible values of between $1-G_X=0.521$ and $-1-G_X=-1.479$. When Π^K is assessed for some combination of both taxes and benefits, its possible range becomes effectively infinite since the concentration curve for such a bundle of both taxes and benefits is not bounded in any directions. For instance, the net benefit share of the bottom 10% of units of some net benefit system will exceed 100% if the subsequent 90% of the population are net tax payers.

From Table 2.4, we note that global progressivity, as measured by the value of Π^K (minus the value of Π^K for transfers), is highest for the FIS and SB programmes combined but lowest for the combination of income tax and NIC. "Various observed FES benefits" and housing transfers appear quite progressive. As indicated above, the Π^K of the combination of all taxes and benefits is not bounded either by $1-G_X$ or by $-1-G_X$ since units may be allocated a cumulative share of some net taxes or benefits that exceeds 1 or that is negative. Actual redistribution (as measured by Π^{RS}) is, however, also related to the size g_i , which makes the group "various benefits" by far the most redistributive, but nevertheless leaves income tax and NIC close to the bottom of the redistributive scale, contributing as they do to no more than 12% of the fall of 0.208 in the Gini

¹⁵ Note that the converse is not true. A Kakwani index of tax (or benefit) progressivity for which $I_{T,X} < G_X$ (or $I_{T,X} > G_X$) does not necessarily imply a redistribution of income as measured by the Gini coefficient of net incomes. Indeed, because of the possible reranking of individuals and as noted above, the finding of a Π^K (or AIR) index that appears to suggest progressivity or redistribution may in fact be followed by the discovery that net incomes, once appropriately reranked, are in fact less equally distributed than in the absence of the tax or benefit.

coefficient. Housing transfers, with an average benefit rate five times smaller than the average income tax and NIC rate, manages to generate almost twice the vertical equity power of the combined income taxes and National Insurance contributions.¹⁶

A proposal that would (unrealistically?) abolish all transfers, leave constant the progressivity of income taxation and NIC but decrease their average rate of taxation to leave constant the net tax/benefit yield to the state would in fact lead to no more than a 0.006 fall in the G_x level. Trying to achieve the model's fall in the Gini coefficient by abolishing all transfers but instituting a "basic income" scheme (or negative income tax system with a single rate throughout the gross income range) would require a uniform tax rate of 43% on all incomes (ignoring of course the effects of any behavioural change in, say, labour supply).

Vertical equity in 1985 Britain (as assessed by Π^{RS}) is thus mostly a matter of the benefit system. Table 2.5 splits the broad elements of Table 2.4 into some of their components. Such decomposition shows that, whilst the transfer system remains the most potently progressive (with FIS and SB again ranking the highest, just above housing benefits), several components of personal income taxation are globally *regressive*. Such is the case of the decrease in the age allowance, the composite rate scheme and the independent taxation of spouses' earned incomes. When weighted by the average tax/benefit rate, such measures show that (1) various observed FES benefits -- including several disability and injury benefits

¹⁶ Not attempting to correct for some FES sample irregularities by failing to incorporate the varying grossing-up weights introduced earlier changes the results very little.

--, (2) the National Insurance basic pensions, (3) FIS and SB, and (4) housing benefits are all individually more redistributive than personal income taxation. NIC and MITR had no overall redistributive effect, as measured of course by the Reynolds-Smolensky index. The composite rate scheme described above counteracts completely the minor redistributive impact of Supplementary Housing Benefits.

How much of the redistribution suggested by Tables 2.4 and 2.5 is in fact "undone" by the violation of principles of horizontal inequity? That is, by how much does AIR differ from AIR^* when we consider the element of the class AIR^* for which $W(p)=-2p$? Table 2.6 displays the elements of the second set of brackets of equation (37). It indicates the value of $I_{N,X-\Sigma T}$ as the T 's are cumulated along the roots of the tree diagram. In parentheses, we indicate the % change in $I_{N,X-\Sigma T}$ as we move down one root, measured relative to the total fall in G_X suggested in Table 2.4 ¹⁷. The classes of taxes and benefits are as in Table 2.4 :

I= CB+OPB+UNB+BP+various observed FES benefits, among which feature many disability and injury transfers

II=NIC+income tax, net of MITR

III=FIS+SB, net of housing component

IV=All housing dependent benefits

For instance, noting that $I_{N,X}=0.271$, we see that adding the class of programmes I to X yields $I_{N,X+I}=0.282$ and causes an increase of 0.011 towards G_N , this increase representing 5.3% of the fall of 0.208 predicted by Tables 2.4 and 2.5.

We first note that horizontal inequity introduces a total bias of 0.018

¹⁷ That is, relative to $G_X-I_{N,X}=0.208$.

between $I_{N,N}=G_N=0.289$ and $I_{N,X}=0.271$. Hence, the reranking of individuals leads to an index of horizontal inequity equal to 8.7% of the index value of total vertical equity. This amounts to almost the degree of vertical equity exerted by personal income taxation and NIC combined, and to the size of the redistribution originally effected by CB and OPB.

A closer look at Table 2.6 also reveals that NIC and personal income taxes appear to contribute very little to total horizontal inequity, and so to some extent do FIS and SB¹⁸. The chief culprits of horizontal inequity seem to be those of class I (various transfers, including CB, OPB, UNB, BP) and class IV (housing-dependent benefits), regardless of the order in which the classes of taxes and benefits are combined. This is not surprising, of course, since the class I and IV benefits are mostly made of categorical grants, i.e., ones for which the state professes to have sufficient independent efficiency or social reasons to make their allocation dependent on factors other than the current equivalent incomes of units. It is also the case that the class I and IV benefits are those that appear to achieve the greatest level of vertical equity, and one would therefore also expect them to generate in the process a more considerable degree of horizontal inequity.

Conclusion

We have, in this chapter, presented tools that may help depict the individual effect of taxes and benefits on the level of progressivity, redistribution and equity exerted by existing or proposed tax and benefit systems. We introduced measures of local progression and showed how we might split total

¹⁸ That SB does not appear to rerank individuals would strongly depend on the type of equivalence scale adopted.

progressivity into the contribution of individual tax and benefit elements. Some elements are regressive over some ranges of gross incomes, and net income tax generally shows a monotonic loss of progressivity. Using a social welfare function with a constant degree of "inequality aversion", we are able to provide an estimate of the extent to which additional government revenues may be collected by a progressive system without a change in the level of social welfare. By such standards and for "middle" values of inequality aversion parameters, the relatively progressive 1985 system could thus have implicitly yielded revenues of between 7% and 35% in excess of those generated by a purely proportional system. Through a similar procedure, it is also possible to produce parameter values of a constant residual progression system that approximates the unequally progressive real systems.

We finally turned to measures of global progressivity and redistribution. We focused our analysis on a general class of indices of redistribution AIR^* which obey, among other principles, the axiom of anonymity in the treatment of post-tax incomes. We noted how the difference between AIR^* and a class of indices of vertical equity, AIR , can lead to a class of indices of horizontal inequity. Combining indices of true redistribution, AIR^* , and indices of horizontal inequity, AIR^*-AIR , can make it possible to judge the desirability of redistributive plans. Choosing an element of the classes AIR^* and AIR , we show how the separate contribution of taxes and benefits to redistribution and equity may be highlighted. An illustration using the 1985 British system reveals that, as expected, benefits targeted to the worse off are the most progressive, whereas income taxes and social security contributions contribute very little to the total vertical redistribution

exerted by the tax and benefit system. The reranking of individuals produces an index of horizontal inequity equal to about 9% of the index value of total vertical equity. Housing benefits and categorical transfers roughly appear to be the main causes of such horizontal inequity, although a more definite empirical conclusion would require, among other things, a more comprehensive definition of original and final income.

Table 2.1

**Progressivity of the Income Tax and of the Global Budget
Constraint**

Family of Four, Owner-Occupiers

Gross incomes	ARP ¹ Y tax	ARP Net	RP Y tax	RP Net	LP Y tax	LP Net
40	45.58	641.94	0.85	0	0	-0.64
120	-1.13	9.84	1.01	0.87	0	2.91
200	8.67	9.30	0.80	0.77	2.37	1.91
280	4.43	4.74	0.85	0.82	1.70	1.52
360	2.68	0.84	0.88	0.96	1.47	1.11

¹ ARP is inflated by a factor of 10⁴ in all tables.

Table 2.2

Elements of Income Tax Progressivity

Family of Four, Owner-Occupiers

X	Net Income Tax				Elementary Income Tax			MITR			Composite Rate Scheme		
	a	ARP	RP	LP	T	TLP1	TLP	T	TLP1	TLP	T	TLP1	TLP
40	-0.18	45.58	0.85	0	0	0	0	-8.91	55.7	0	1.62	-10.13	0
80	-0.09	11.40	0.92	0	0	0	0	-8.91	13.9	0	1.62	-2.53	0
120	0.014	-1.13	1.01	0	10.22	17.90	0.33	-8.91	6.19	0	0.31	-25.22	-0.30
160	0.083	13.55	0.76	3.61	22.21	10.07	0.35	-8.91	3.48	0	0	0	0
200	0.127	8.67	0.80	2.37	34.22	6.44	0.36	-8.91	2.22	0	0	0	0
240	0.155	6.02	0.83	1.93	46.20	4.48	0.37	-8.91	1.55	0	0	0	0
280	0.176	4.42	0.85	1.70	58.21	3.29	0.38	-8.91	1.13	0	0	0	0
320	0.192	3.39	0.87	1.57	70.22	2.52	0.38	-8.91	0.87	0	0	0	0
360	0.204	2.68	0.88	1.47	82.22	1.99	0.39	-8.91	0.69	0	0	0	0
400	0.213	2.17	0.89	1.41	94.85	4.07	0.52	-9.54	-1.9	-0.1	0	0	0

$$TLP_i = T_i LP_i / N$$

$$TLP1_i = T_i (LP_i - 1) * 10^4 / X^2$$

$$\text{Net ARP} = \sum TLP1_i$$

$$\text{Net RP} = 1/(1-a) - \sum TLP_i$$

Table 2.3
Elements of Net Progressivity
Family of Four, Renting

X	a	ARP	RP	LP	Income Tax		NIC		HB and HBS		FIS and SB	
					TLP1	TLP	TLP1	TLP	TLP1	TLP	TLP1	TLP
40	-1.28	568.9	0	-0.78	-1.31	0	0	0	137.1	0	433.1	0.44
80	-0.21	48.12	0.68	-0.79	-0.32	0	4.19	0.07	44.26	0.07	0	0
120	0.06	58.77	0.25	11.90	19.47	0.32	1.86	0.10	37.44	0.41	0	0
160	0.20	12.00	0.76	1.97	10.95	0.37	1.05	0.11	0	0	0	0
200	0.24	7.68	0.80	1.65	7.01	0.39	0.67	0.12	0	0	0	0
240	0.26	5.33	0.83	1.49	4.87	0.41	0.47	0.12	0	0	0	0
280	0.28	3.92	0.85	1.39	3.58	0.42	0.34	0.13	0	0	0	0
320	0.29	0.41	0.98	1.05	2.74	0.42	-2.33	0	0	0	0	0
360	0.29	0.32	0.98	1.04	2.16	0.42	-1.84	0	0	0	0	0
400	0.29	0.26	0.99	1.04	1.75	0.42	-1.49	0	0	0	0	0

$$TLP_i = T_i LP_i / N$$

$$TLP1_i = T_i (LP_i - 1) * 10^4 / X^2$$

$$\text{Net ARP} = \sum TLP1_i$$

$$\text{Net RP} = 1/(1-a) - \sum TLP_i$$

Table 2.4

Kakwani Indices and Redistribution

Main Tax and Benefit Groups

Varying grossing-up weights

	Various Bene- fits ²	Income Tax and NIC ³	FIS and SB ⁴	Housing Trans- fers ⁵	All taxes and Transfers ⁶
Π^K	-0.886	0.085	-1.325	-0.684	2.779
g_i	-0.132	0.275	-0.017	-0.056	0.070
$g_i / (1 - \sum g_i) * \Pi^K$	0.119	0.025	0.024	0.041	0.208

² Include CB, OPB, UNB, BP, and various observed FES benefits.

³ Net of MITR.

⁴ Net of housing-dependent components.

⁵ Includes MITR and supplementary benefits dependent on housing requirements.

⁶ The Gini coefficient of gross (original) income (using the varying grossing-up weights specified in the text) equals 0.479.

Table 2.5

Kakwani Indices and Redistribution

Detail of Tax and Benefit Elements

Varying grossing-up weights

	Elementary Income Tax	Fall in Age allowance	MITR	Composite Rate Scheme	Independent Taxation	National Insurance Contributions	Various benefits ⁷
Π^k	0.118	-0.067	-0.006	-0.755	0.488	0.009	-0.844
g_i	0.210	0.001	-0.024	0.002	-0.001	0.065	-0.053
$g_i / (1 - \sum g_i) * \Pi^k$	0.027	0	0	-0.002	-0.001	0	0.048

	CB and OPB	Unemploy- ment Benefits	Basic Pension	FIS and SB	Housing Benefits: Rents	Housing Benefits: Rates	Supplementary Housing Benefits
Π^k	-0.539	-0.674	-1.034	-1.314	-1.188	-1.171	-1.109
g_i	-0.031	-0.002	-0.046	-0.022	-0.019	-0.008	-0.001
$g_i / (1 - \sum g_i) * \Pi^k$	0.018	0.001	0.051	0.031	0.024	0.010	0.001

⁷ Various observed FES benefits.

Table 2.6
Horizontal Inequity and the British Tax and Benefit System
Varying grossing-up weights

$G_X = 0.479$

$I_{N,X} = 0.271$

I 0.282 (5.3)			II 0.271 (0.0)			III 0.272 (0.5)			IV 0.274 (1.4)		
II 0.284 (1.0)	III 0.282 (0.0)	IV 0.285 (1.4)	I 0.284 (6.3)	III 0.272 (0.5)	IV 0.274 (1.4)	I 0.282 (4.8)	II 0.272 (0.0)	IV 0.274 (1.0)	I 0.285 (6.3)	II 0.274 (0.0)	III 0.274 (0.0)
IV 0.288 (1.9)	IV 0.287 (2.4)	III 0.287 (1.0)	IV 0.288 (1.9)	IV 0.274 (1.0)	III 0.274 (0.0)	IV 0.287 (2.4)	IV 0.274 (1.0)	II 0.274 (0.0)	III 0.287 (1.0)	III 0.274 (0.0)	II 0.274 (0.0)
III 0.284 (0.0)	II 0.284 (1.0)	II 0.288 (1.4)	III 0.284 (0)	I 0.284 (5.8)	I 0.288 (6.7)	II 0.284 (1.0)	I 0.284 (5.8)	I 0.287 (6.3)	II 0.288 (1.4)	I 0.288 (6.7)	I 0.287 (6.3)
III 0.289 (0.5)	II 0.289 (1.0)	II 0.289 (1.0)	III 0.289 (0.5)	I 0.289 (7.2)	I 0.289 (7.2)	II 0.289 (1.0)	I 0.289 (7.2)	I 0.289 (7.2)	II 0.289 (1.0)	I 0.289 (7.2)	I 0.289 (7.2)
IV 0.289 (2.4)	IV 0.289 (2.4)	III 0.289 (0.5)	IV 0.289 (2.4)	IV 0.289 (2.4)	III 0.289 (0.5)	IV 0.289 (2.4)	IV 0.289 (2.4)	II 0.289 (1.0)	III 0.289 (0.5)	III 0.289 (0.5)	II 0.289 (1.0)

I= CB+OPB+UNB+BP+various observed FES benefits

II=NIC+income tax, net of MITR

III=FIS+SB, net of housing component

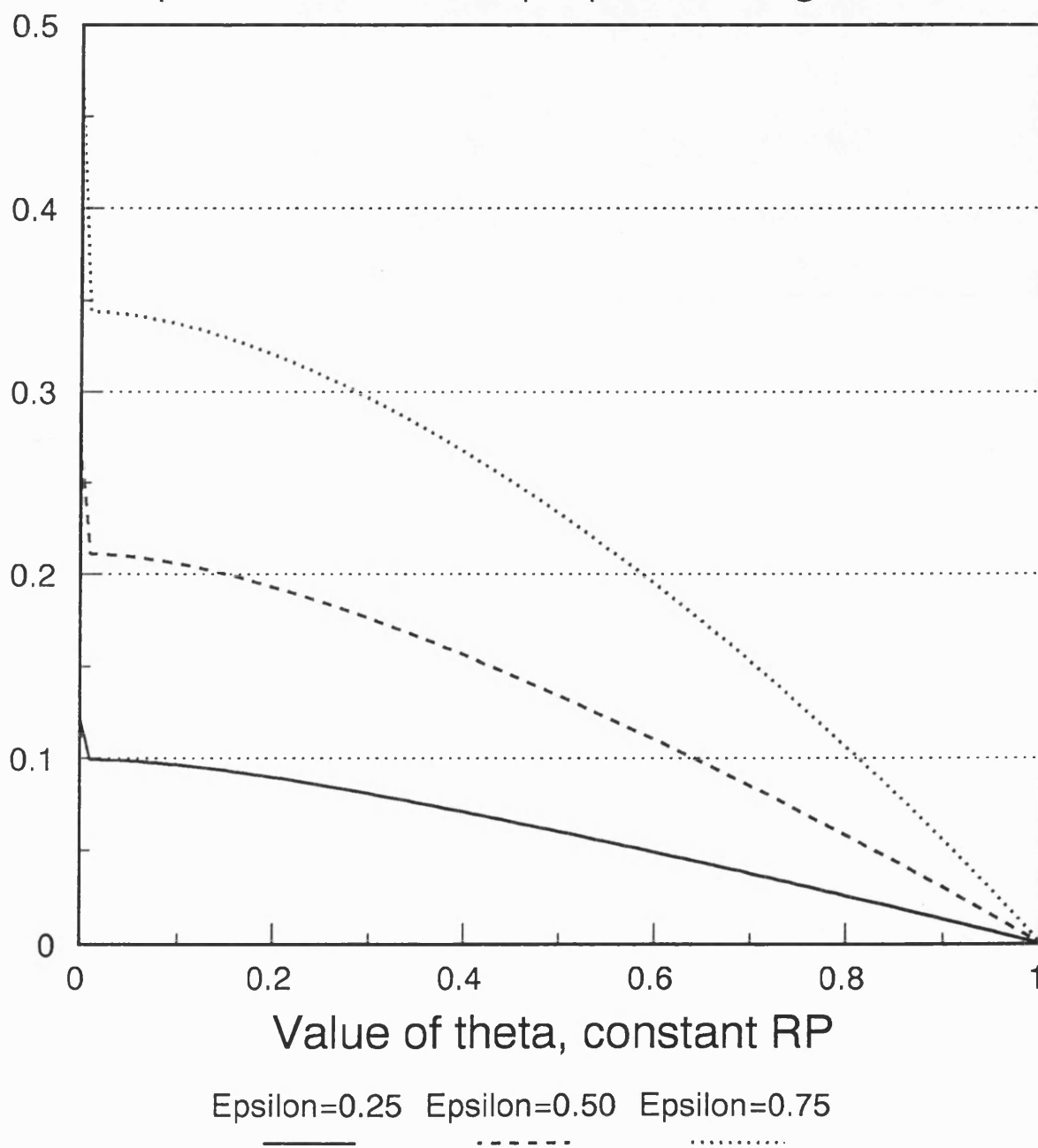
IV=All housing dependent benefits

The table indicates $I_{N,X-\Sigma T}$ where the T_i are cumulated down along the tree roots; parentheses show the % change in $I_{N,X-\Sigma T}$ relative to $(G_X - I_{N,X}) = 0.208$, as one moves downwards. $G_N = 0.289$.

Figure 2.1

**Proportional vs constant RP tax/ben system
For equivalent social welfare**

RP surplus tax revenue, proportion of gross Y



Chapter III: Understanding the Take-up of State

Benefits

Introduction

There are three major justifications for attempting to further our understanding of the take-up of state benefits.

First, the institution of "safety nets" is at the centre of policy discussions, especially in the context of the liberalisation of former command economies. As Atkinson (1992) points out, these safety nets are usually described as forming essential elements of "transition" processes, both in order to reduce the harsh blow of liberalisation on the newly poor and unemployed and to maintain the political and social will to reform. The nature of such safety nets is, however, rarely discussed, and the social and economic effectiveness of state benefits is normally taken for granted.

Second, the empirical analysis of the effect of taxes and benefits on the microeconomy and, in particular, on households and consumers is now a widespread aid of economic policy¹. Indeed, it is now unimaginable to consider major tax and benefit reforms in most developed economies without detailed microsimulation exercises. As we have seen, the general aim of the exercises is to convey useful information on the efficacy of government intervention, the inferences relying particularly on the use of micro data and on the more or less uniform application of alternative sets of tax and benefit regulations across a

¹ We have already suggested in the first two chapters some methodological and empirical features of such tax and benefit analysis.

sample. It should, however, be recognised that the government's discretion in applying such rules will be limited, on the one hand, by tax evasion and avoidance and, on the other hand, by a less than full take-up of state support. The risk therefore exists that an unsuspicious tax/benefit analyst will overstate the plausible effects of any policy simulation (including the value of indices of tax and benefit progressivity and redistribution), for not all targeted agents will in practice be as affected by policies as proposals would otherwise intend. That tax and benefit policy does not in practice generate the results originally intended can also provide clear cases of horizontal inequity, in addition to those measured in the previous chapter. Moreover, even if all did receive the intended support and none did try to dodge the imposition of taxes, the imperfection of micro data -- in the form of errors and biases inherent to the use of micro data -- would still raise the issues of the validity and accuracy of the normative and positive observations flowing from the tax and benefit analyses.

Third, the intervention of the government always comes at some cost, some of which may not easily emerge from the comparative analysis of competing tax and benefit proposals. Among the various costs involved figure: the expenses of implementing and administering the proposed reforms; the opportunity costs of not using resources to other ends (e.g, substituting social security payments for the provision of public goods); and the deadweight welfare losses stemming from the distortion of prices by all reasonable tax and benefit packages, as reviewed for instance by Hausman (1985) in the case of the effect of taxes on the choice of labour supply. Less documented than the latter, but probably no less important, are the costs willingly incurred by units to avoid the imposition of taxes or to

avail themselves of the government's support. These latter costs may usefully be seen as contracting costs between the government agency wishing to distribute help -- in order, say, to maximise a social welfare function or to provide a social safety net -- and the population units, about whom the government agency has imperfect information, who face uncertainty of benefit entitlement, and who must usually bear at least some of the contracting costs of claiming a state benefit.

We divide our investigation of the take-up of state benefits into two chapters. In this one, we present some of the concepts and the possible structure of an analysis of the take-up of state benefits. We first review some previous work on the topic and we discuss the role of imperfect information and uncertainty in the modelling of benefit take-up. We then impose some simplifying assumptions to make our analysis more easily implementable, and we consider an application to the take-up of Supplementary Benefits in Britain, pondering in the process some of the computational challenges.

The second part of our study is presented in Chapter IV and applies the work of Chapter III to some Family Expenditure Survey data gathered on the take-up of Supplementary Benefits in 1985 Britain. We discuss and model some of the suspected deficiencies of such survey data. We also consider the grossing-up of our sample and the effect of differential sampling upon the validity of our econometric exercise. We then present the results of our estimation over the sample, examining in particular the estimated "modelling" and administrative entitlement errors and "costs to claiming". An enquiry into the statistics of benefit take-up follows.

A- The Concepts

1- A Review

Our enquiry is not, of course, the first to consider the determinants and the consequences of the imperfect take-up of state support or -- in the American terminology -- the less than full rate of participation in welfare programmes. We start by reviewing the economic and econometric contributions and weaknesses of some of these studies, using these as a preamble to many of the issues that will come up in the course of our own enquiry. For instance and except for the American investigations, all of the studies reviewed here acknowledged, in one way or another, the empirical difficulties of assessing the apparent failures to claim the support to which units are apparently entitled. We begin by reviewing quickly some features of studies of benefit take-up in "continental" Europe, then note in more detail the econometric contributions (mainly on labour market behaviour) of American enquiries on the availability of welfare programmes, and finally examine the evolution of and the state of take-up research in the U.K..

Dick (1986) reports a less than 50% take-up rate of German housing allowances in the early 1970's. His estimate depends on the combination of data from two fairly different sources, one comprising official statistics on aggregate receipts of housing allowances, and the other providing sample data on the population that are then used to predict the size of the set of those eligible to such allowances. A number of approximations are necessary to predict eligibility, and grossing-up variations add to the uncertainty surrounding the final value of the take-up estimate.

Nyman and Schwarz (1991) report similar data difficulties in assessing the

take-up of Swedish housing allowances, which they do also by combining information from two different data sources. A rather indirect benefit of using such complementary data sets is, however, that the temptation of making the 'relevance' of official receipts dependent on eligibility as **computed** by the take-up analyst is lesser since the eligibility of actual recipients either cannot be assessed (only their total number is known) or such eligibility is simply then taken as that appraised by the agency responsible for the administration of the state benefit (viz, given by the administrative data source). As we will see in this and the following chapter, this methodological subtlety can make a large difference in the nature of the final take-up results and influence greatly the positive and normative conclusions reached on the efficiency and efficacy of state support. Among other things, Nyman and Schwarz also report that take-up naturally seems to increase as the particular benefit becomes better known and better accepted.

In a 1989 report the Centre d'Etudes des Revenus et des Coûts discusses the claims of French widows' pensions and finds that a strong increase in the proportion of those in receipt of such pensions could be noted between the periods of 7 and 19 months following widowhood. The ultimate widows' pensions take-up rate of 84% was computed very carefully on the basis of a special enquiry which allowed the inclusion in the sample of the apparently eligible but not in receipt only those for which a positive entitlement could be established -- for all purposes -- as certain. Hence, contrary to the presumption applicable to many of the British enquiries reviewed below, this take-up estimate, if anything, probably overestimates the actual one. Remaining failures to claim could typically be traced

to a lack of ready information on claiming conditions and procedures. On precisely this, Strauss (1977) finds unsurprisingly "that the availability of information on eligibility determination affects participation in a social welfare programme" (p.395). He bases his empirical findings on the analysis of the receipts of Supplemental Security Income in North Carolina in 1974, for which he runs simple linear regressions on total enrolment data of a sample of 100 counties.

Ashenfelter (1983), as for the following American investigations, treats "participation" in welfare programmes and labour supply behaviour as jointly determined. With the availability, say, of a welfare programme for which eligibility requires that earnings fall below a particular level, some units will not take up the offer of state support and therefore not change their supply of labour, others will "participate" in the programme without having first to become entitled to it (and will therefore not necessarily have to change their level of labour market earnings), and others will change their labour supply in order to take up the offer of state support. One can therefore see that in determining welfare programme participation and expenses, labour market behaviour and incentives will matter not only for those who would be eligible to the programme in the first place, but also for those who would find it suitable to change their labour supply in order to receive state support. Using data from the Seattle and Denver Income Maintenance Experiments and modelling the presence of participation costs, Ashenfelter's results "suggest that differences in participation across negative income tax plans are due primarily to differences in program breakevens or generosity. Tax rate variations [and thus marginal labour supply incentives] have only small additional effects on participation" (p.524). In particular, compensated

labour supply wage elasticities are found to be statistically insignificant. More importantly (and more surprisingly), the receipt of welfare seems not to be affected by the presence of costs to claiming welfare or to participating in the programme; this, however, is a purely suggestive finding since no test statistics on such an hypothesis is provided that can be interpreted unambiguously. It is moreover no easier to find a suitable economic interpretation of the error term in the Probit estimations: is it that income and entitlement are not measured accurately? Are there participating units with income that is apparently too large to be consistent with programme eligibility?

The explicit consideration of take-up costs and welfare stigma where earnings and programme eligibility are treated as endogenous is also a feature of Moffit (1983). The agents' utility function is discounted by the size of the fixed and variable (with the benefit level) costs that are incurred with the receipt of state support. It follows that, in such a context, "the labour supply response to welfare programmes depends upon the stigma of welfare benefits" (p.1024). Using observed labour supply behaviour and receipts by lone mothers of the American Aid to Families with Dependent Children (AFDC), and estimating the value of some tax and benefit parameters, Moffit reports values of participation burdens that suggest a strong level of fixed cost (or stigma) but estimates a variable burden to claiming that is not significantly different from zero. Falls in marginal rates of benefit withdrawal would generate some increase in labour supply by those already eligible to the programme but they would also encourage a substantial fall in hours of work by those who would then wish to become entitled to the state's offer of support. Finally, by decreasing the utility value of

welfare, stigma and other participation costs naturally decrease the labour supply disincentives of such welfare programmes.

One of the most elaborate American econometric enquiries on the links between labour market behaviour and applications to and receipts of state support is that of Halpern and Hausman (1986). They study the choice of applying for Disability Insurance (DI) in the U.S. when there is uncertainty about the outcome of the application process and when labour supply is also endogenous. There are three possibilities: an agent does not find it worthwhile to apply for DI and continues working or not working as he used to; an agent applies for and is awarded DI (and can then retire); an agent applies for but is not granted DI, and must subsequently face a lower wage if he then chooses to participate in the labour market. Agents base their decision in the maximisation of their expected utility. An application for DI is made when the expected utility benefit of a successful request outweighs the expected utility cost of the fall in the market wage when a DI application is unsuccessful. Their analysis will thus shortly show to be conceptually similar to our own. A sophisticated stochastic specification is adopted [inspired partly by Burtless and Hausman (1978)] which leads to the identification of various distributional parameters through the joint observation of labour supply behaviour and application choice (both revealing information on the work/leisure preferences of agents). Their "estimates indicate that the applications decision is a good deal more sensitive to benefit levels than to the probability of acceptance" (p.158), suggesting that the lower wage following an application rejection is not a strong deterrent to applying. They do not detect a significant empirical presence of fixed costs to applying for DI, although

application rules and receipt delays clearly suggest that there are ought to be some.

Empirical British studies do not unfortunately fare very well in comparison to such rather elaborate American work on the take-up of state support. A review of British work on the subject, belonging to various social sciences, can be found in Craig (1991), who also provides a useful discussion of some of the important methodological and conceptual issues inherent to the study of the take-up of state benefits. Apart from the scarcity of studies relating the choice of claiming state support to labour market participation, British enquiries have also often neglected some important methodological issues, the most important of which being whether the use of imperfect data to predict eligibility may explain the computation of less than full benefit take-up rates.

One of the earliest econometric studies (in the United Kingdom at least) appears to be that of Altmann (1981), who maximises a simple logit likelihood function of the take-up of SB among male pensioners, using FES data of several years (thus making possible a comparison of the results across time). She finds that, "contrary to results of all previous studies", "the amount of SB to which one is entitled does not influence take-up" (p.19). More recent studies are those of Fry and Stark (1987) and Blundell, Fry and Walker (1987). The first one investigates the claims of SB, while the second examines the take-up of housing benefits in the UK. Fry and Stark (1992) have applied the same framework to monitor the evolution of take-up in a period (the late 1980's) in which the British system of state benefits underwent some important changes. All studies are based on a probit model with one source of randomness. They exclude or provide separate

analyses for substantial subsamples, such as those of the self-employed and the pensioners. The likely presence of modelling errors and entitlement discrepancies is mentioned but it does not feature in the analysis. It is not obvious how to extend their use of the logarithm of the calculated entitlement level to yield estimates of the costs that may be involved in taking up a benefit.

Dorsett and Heady (1991) have recently extended the above analysis to try modelling the interaction between the take-up of benefits, focusing on the link between the claims of Family Income Supplement and those of Housing Benefits. The computation by the take-up analyst of entitlement levels to such benefits is, however, even more uncertain and subject to errors than the assessment of eligibility to Supplementary Benefits or Income Support (say), although again no account is taken of the presence of such errors in the derivation of take-up estimates and determinants. On this, Craig (1991) particularly suggests that a "priority" for further research should be "the refinement of strategies for identifying eligible non-recipients" (p.559). This is so since, as he notes,

"Even where calculation of income and eligibility is based on very carefully designed research instruments, the divergence between research and administrative assessments means that the first cannot be taken at face value" (p.560).

As we shall see below, this necessary refinement shall be one of the features of our own enquiry.

2- Incomplete Take-up and the Costs of Claiming

Various reasons can thus be given to explain the observed incompleteness of the take-up of state benefits. The first one is that the deficiency of claims may be simply a consequence of using and computing imperfect data on benefit

entitlement. Figure 3.1 illustrates this point. There we display an imaginary population set for which those receiving a state benefit are indicated by a "c" and those not receiving, by a "nc". Two sets of eligibility are shown, the "true" one and the "calculated" one. For simplicity, the truly eligible can be for now understood as those from whom a benefit request to the government agency would have been honoured². Calculated eligibility is assessed on the basis of an analyst's micro data and on his understanding of the benefit regulations. Figure 3.1 shows that for both sets of eligibility the number of eligible is 10, and that the true take-up rate in this example would equal 100%: everyone of the 10 truly eligible agents receives the benefit. An assessment of take-up on the basis of calculated eligibility, excluding those not thought to be eligible, would, however, suggest a 70% take-up rate. Imperfect data and errors of computations have indirectly led to the rejection of three truly entitled units from take-up calculations, and we have wrongly included three ineligible units. Alternatively, an aggregate take-up estimate based on the number of observed claimants divided by the number of those computed to be eligible would be equal to 100%. The take-up rate is thus only underestimated if one discards those are in receipt in spite of not being thought to be eligible. This suggests that a failure to beware the imperfect use of imperfect data may lead to significant biases in the estimation of the efficacy of state benefits.

A second factor put forward to explain incomplete take-up rates is that non-claimants behave irrationally. Alternatively, agents refuse to claim their benefit entitlement for social and psychological reasons which apparently fall

² We will discuss this and related issues in detail later.

outside the domain of economics. Notwithstanding this, we shall maintain that utility theory and rational behaviour alone are perfectly able to explain decisions not to claim a state benefit. Information about the application procedure and the administration of the state benefit may not be freely and perfectly available. The material costs (in time, transportation, correspondence, etc.) of claiming may simply outweigh the value of the state support. The perceived non-monetary disutility of receiving state support might not be compensated by the size of the benefit. This disutility may stem from a sense of personal guilt for being a "burden" to society. It may also arise from an analogous loss of pride and self-esteem, from social stigma, or from the potential revelation to others (e.g., to potential employers) of undesirable "signals" and characteristics. There may be uncertainty about the value of one's entitlement, which, in the presence of risk aversion, could tend to make it less worthwhile to request an uncertain level of state support. All this is summarised in the following quote from a report by the Supplementary Benefits Commission (1978) taken from Craig (1991, p.543):

"All we can say is that this reluctance to claim appears to come from some mixture of pride, ignorance, a sense of stigma, reluctance to make the efforts a claim calls for, a desire for self-sufficiency on the part of an individual or family, an unwillingness to become involved with a government agency and a feeling that the whole business is not worthwhile."

3- Imperfect Information, Uncertainty, and Unobservability

There are at least three "participants" appearing in any endeavour to analyse the take-up of a state benefit. They are the set of population units (e.g., families or households who may or may not receive the benefit), the government agency responsible for the administration of the state benefit, and the take-up

analyst. The government institutes a set of rules which establish the true entitlement of the population units to some state support. We may or may not wish to believe that the government, in establishing these rules, has been able to legislate the set of entitlements leading to its preferred social optimum. The units then consider whether it is worth their while to request the benefit. The government agency has imperfect information on the characteristics of the units and therefore on their true entitlement to the benefit. It is also liable to make administrative errors. The units have imperfect information and feel uncertain both about their entitlement as established by the set of benefit regulations and about the ability of the government agency to establish that true (legislated) entitlement. Units may also feel uncertain about the "hassle" involved in requesting the state benefit. Through forming opinions about each other these first two participants thus determine one of the following four outcomes: a successful benefit request by a unit, an unsuccessful one, a benefit request that would have been successful had it been made (it was not), and a benefit request that was not made and that would not have been granted even if it had been made.

The take-up analyst may not be able to observe separately each of these four outcomes; he (or she!) might, for instance, only know whether or not a successful claim is registered by a unit. Besides, the analyst does not observe with perfection the characteristics of the population units. He cannot assess with complete accuracy the costs involved in requesting a state benefit and he generally cannot monitor directly the *ex ante* private perception of the state benefit. Neither can he ascertain exactly the units' true entitlement, nor is it always possible to observe the value of the units' benefit entitlement as assessed by the government

agency.

We now try to characterise these concepts a bit more formally. We define B^* as the true entitlement of a unit to some state support. B^* may not designate the socially optimal level of entitlement on which all social policy makers agree, but it is by construction the one which the government wishes to establish and which it officially publicises. Because of imperfect information on potential state benefit recipients and due to administrative errors, the actual level of entitlement as assessed by the administrative agency will, however, generally differ from B^* and will equal

$$B_g = B^* + \epsilon_g \quad (1)$$

ϵ_g is a stochastic error term whose mean does not have to be zero. It subsumes the structural and random deviations of B_g from B^* . Because of the presence of imperfect information and administrative lapses, the evaluation of eligibility based on B^* may differ from that made on B_g . This will cause what are known as type I and II errors on the part of the government agency. A type I error occurs when the application of a truly eligible unit is wrongly rejected; the probability of this happening, given that a unit is truly entitled, is given by

$$\text{Type I error: } P(B_g \leq 0 \mid B^* > 0) \quad (2)$$

Conversely, a type II error results when a unit which is not truly entitled nevertheless fruitfully requests the state's support. The conditional probability of this arising is:

$$\text{Type II error: } P(B_g > 0 \mid B^* \leq 0) \quad (3)$$

In deciding whether to ask for the benefit, units make their own private assessment of the B_g that would be attributed to them if they were to make the request. This assessment is described by B_p :

(4)

An important link between B_g and B_p can be expected. It is in the best interest of the population units to seek to pinpoint the distribution of B_g as exactly as possible, for some given level of search costs. In choosing whether to request the benefit, *ceteris paribus*, a unit will find it desirable to reduce the uncertainty of B_p (unless one is a risk lover) and to keep it as closely as possible to the government valuation B_g . This will generally control the chance of one's application being rejected if a request is made³, the chance of one's application not being made when the availability of better information would have made it worthwhile, and the chance of an application being made when it was in fact not worthwhile *ex ante* to do so. Ideally, ϵ_g ought to be a constant known to the unit such that it makes B_p equal with certainty to the particular B_g that is drawn from the distribution of $B^* + \epsilon_g$.⁴

For now, however, we allow for the possibility that private agents may

³ The possibility that a reassessment of B_g be performed if a unit feels that its benefit request was unfairly rejected would influence the distribution of B_g but would not otherwise affect the discussion here.

⁴ This is what we shall assume later for the purposes of our empirical investigation.

have a view B_p of B_g that be very different from the entitlement value assessed by the government agency. Units may, for instance, systematically overestimate or underestimate the average of B_g , or exaggerate or not its variability. The probability that some type I and II errors (defined above) may result once a benefit request is made will influence the wish of the units to seek the state benefit, unless of course the units have no previous knowledge or suspicion that such faults may occur.

4- Requesting a State Benefit

Our income units are faced with a discrete choice of requesting or not requesting a benefit which is offered by the state. We model that choice as a problem of maximising expected utility: if the expected utility of requesting the state's support is greater than that of not doing so, a unit will choose to ask for the benefit, otherwise it will not. The utility function is $U(N, Q)$, where N is net income and Q is "quietness", "tranquillity", or absence of "hassle". If the unit chooses to claim, it enjoys utility $U(N^0 + B_p^b, Q' - E_p - E_a)$, with $Q' - E_p - E_a$ the "tranquillity" enjoyed if a claim is made. If it chooses not to claim, the unit has utility $U(N^0, Q^0)$ with complete certainty. N^0 and Q^0 are the levels of net income and "quietness" when no state support is requested. Thus $Q' - E_p - E_a$ and Q^0 differ by the extent to which a unit suffers a loss of quietness and tranquillity from the act of claiming a state benefit⁵. B_p^b -- where the b superscript distinguishes perceived benefit

⁵ The choice of applying or not for state support offers clear similarities with the decision to evade tax or not, where such evasion generates a probability of detection by the state and the payment by the evader of an expected fine. Evasion will this occur when the expected utility of "cheating" the government exceeds that of remaining perfectly "honest". Empirical and theoretical work on such tax evasion is reviewed in Cowell (1990).

payable B_p^b from perceived entitlement B_p -- is defined as follows:

$$B_p^b = \begin{cases} 0, & \text{if } B^* + \epsilon_p = B_p \leq 0 \\ B^* + \epsilon_p, & \text{if } B^* + \epsilon_p = B_p > 0 \end{cases} \quad (5)$$

This says that no perceived benefit, B_p^b , is associated with a perceived non-positive entitlement, and that privately perceived benefit equals privately perceived entitlement when the latter is positive. We assume that $Q' - E_p - E_a$ is always lower than Q^0 : net gains in utility from requesting the state's support can only come from the grant of a benefit. There may exist positive feelings of "fulfilment of duty" or other benevolent sensations in requesting a state benefit, but these would plausibly never prevail on the negative ones and on the contracting costs of making the request. E_p is the level of the loss of Q which units may not know in advance of seeking the benefit, reflecting among other things the uncertainty of potential claimants about the hassle of the claiming procedures. Finally, E_a , although known to the units, is unknown to the take-up analyst and exhibits the extent to which there may be unobservable individual "cost" characteristics.

Thus, in deciding whether to seek a state benefit and for a given B^* and E_a , a unit must compare the expected utility of a request with that of the *status quo*:

$$\int_{E_p} \int_{\epsilon_p} U(N^0 + B_p^b, Q' - E_p - E_a) f(\epsilon_p, E_p) d\epsilon_p dE_p \stackrel{?}{\geq} U(N^0, Q^0) \quad (6)$$

with B_p^b as defined above and $f(\epsilon_p, E_p)$ being the distribution of ϵ_p and E_p as assessed by the unit. We characterise the decision to apply for the benefit by D , such that, given B^* and E_a ,

$$(D|B^*, E_a)=1, \text{ if } \int \int_{E_p, e_p} U(N^0 + B_p^b, Q' - E_p - E_a) f(e_p, E_p | e_a, E_a) de_p dE_p > U(N^0, Q^0) \quad (7)$$

$$(D|B^*, E_a)=0, \text{ otherwise}$$

Neither B^* nor E_a is directly observable. Indirect information on B^* can, however, be gathered by the analyst by computing B_a with

$$B_a = B^* + \epsilon_a \quad (8)$$

ϵ_a shows the discrepancy between the analyst's appraisal of units' entitlement and the true measure B^* . Such errors will stem from sampling defects (e.g., incomplete data, presence of measurement and reporting errors), from faults in using the sample data, from changing family circumstances between the period of a benefit request and that of the survey, and from other inaccuracies in the assignment by our computer model of benefit entitlement.

One of the other likely constraints to the analyst is that, besides not being able to observe B^* and E_a directly, he is only aware of the records of successful claims. We denote the grant of a benefit to an applicant unit by A , such that $A=1$ if eligibility is granted ($B_g > 0$) by the government agency, and $A=0$ otherwise. For a receipt to be recorded, we must then have:

$$(D=1) \cap (A=1) \quad (9)$$

Given B^* and E_a , the probability of such a receipt is:

$$P(D=1 \cap A=1 \mid B^*, E_a) = \begin{cases} P(B_g > 0), & \text{if } (D|B^*, E_a)=1 \\ 0, & \text{if } (D|B^*, E_a)=0 \end{cases} \quad (10)$$

Hence, the record of rather few receipts of a state benefit can be due to two factors. Firstly, units may consider it not worthwhile to request the benefit. They may, for instance, have an overly dismal view of the offer of state support, they may face large inconveniences to claiming, or they may be strongly risk averse in the presence of entitlement uncertainty. Secondly, those requesting the benefit may not be deemed (wrongly or rightly) to be eligible by the government agency⁶. In the absence of inconveniences to seeking the benefit, the closer to and the less variable around B_g B_p is, the better the match between benefit requests and successful receipts.

Were one of the government's aims be to increase the rate of take-up without changing the entitlement rules or the administration of the benefit, the only option left would then be to maximise the number of valid (to the agency) applicants. This could be achieved in two ways. Costs to claiming might first be decreased. Second, the government could generate a more positive perception among the units of the benefit's generosity, though an unreasonably positive view would distort the units' decision, would decrease their expected welfare, would result in an overly high level of application rejections, and would probably not be sustainable for very long. It must also be said that regardless of benefit expectations and as long as B_g is not perfectly known *ex ante* to the units, there

⁶ Another factor, which will pop up later, may be that units sometimes fail to report faithfully to an interviewer their receipt of a state benefit.

will always exist some units seeking the benefit whose application is rejected by the agency, as well as others who do not make a request but who would nonetheless be deemed eligible. This is in addition to the presence of the type I and II errors discussed previously, which concern the position of B_g around the true level of entitlement B^* .

Understanding the take-up of state benefit then raises the econometric problem of evaluating the probability that a unit successfully claims the benefit when E_a and B^* are not directly observable, but when only $B_a = B^* + \epsilon_a$ is known (i.e., can be calculated). That probability can be expressed as:

$$P(\text{successful claim}) = \int \int_{E_a, \epsilon_a} P(D=1 \cap A=1 \mid B^*, E_a) f(\epsilon_a, E_a) d\epsilon_a dE_a \quad (11)$$

with $B^ = B_a - \epsilon_a$*

As mentioned previously, to be in receipt of a state benefit involves the passing of a "double hurdle". First, a unit must consider it worthwhile to seek the state's support and, second, the requesting unit must also be deemed eligible by the government agency. Equation (11) simply computes the expected value -- over the unobservable E_a and ϵ_a -- of the probability that such a double hurdle is successfully passed, given the value of the parameters of the model and that of the analyst's calculation of entitlement, B_a .

Even in the context of this general model of the take-up of state benefits, it must be said that a few analytical simplifications have already implicitly and inevitably appeared. The only decision modelled in this discussion is that of requesting a state benefit or not. We have ignored, for example, the potentially

interesting links of the likelihood to take up a state benefit with the choice of one's labour supply or savings. It is in principle plausible that the unobservable distribution of inconveniences from claiming be not independent of the stochastic variation in labour supply and original incomes and thus of potential state benefit recipients, since state benefits are often means-tested. An agent who might be more likely to work or to save in excess of what his observable characteristics might predict might also face unusually large costs to taking up government support.

B- An Applied Structure

1- A Simplified Model

There are two major problems with the econometric model as developed up to now. The first one, a parameter identification problem, will remain partly with us throughout the following empirical application. A close look at equation (11) reveals that it is not easy to discern statistically the presence of ϵ_a from that of ϵ_p , and the role of ϵ_a from that of ϵ_g , from the pure observations of receipts and non-receipts of state benefits. This is mostly because we are not able to observe B^* directly and thus cannot ascertain independently the extent of the presence of the analyst's own errors in measuring the units' entitlement. Hence, it is difficult to determine whether variations of B_g from B_a are due to variations in ϵ_p or ϵ_a . When possible, a discrimination of the presence of ϵ_a from that of ϵ_p will rely rather heavily on the specification and nonlinearity of the functional form of the utility function. Similarly, E_a and E_p are not easily statistically distinguishable from each other, and, when distinguishable, the estimated relative parameters of their distribution will depend largely on the specification of the utility function.

The second problem with the present model has to do with the computational difficulty of handling (11) as a basic equation for empirical work. Equation (11) involves the computation of the level of expected utility over the range of two stochastic variables, E_p and ε_p , in order to determine $(D/B^*, E_a)$; what is more, we ought to do this for the whole ranges of E_a and ε_a . In simplifying our model we will focus our attention on the identification of "modelling errors" in assessing the level of family units' entitlement to state support, while neglecting uncertainty of entitlement and of costs to claiming on the part of units. An important feature of the present study will then be to recognise explicitly the presence of modelling inaccuracies in the calculation of the value of some economic variables of interest, and to appraise the role which these errors may have in normative and positive analyses. Hence, we shall make three simplifications, trying at each stage to discuss their implications.

(a) Entitlement Certainty

We first impose the strong restriction that units know B_g^b -- where, again, the superscript b denotes the value of benefit payable by the government and is defined similarly to B_p^b -- with certainty before deciding whether it is worthwhile to request the state benefit. To achieve this, units will plausibly need some awareness of the benefit regulations, a knowledge of the experience of other applicants, and an understanding of the characteristics they could successfully convey to the agency in the event of a benefit request. By this assumption we do not imply that the population units can separately identify B^* and ε_g ; we only assume that the level of benefit that would be granted to a unit by the government agency can be reasonably well assessed, *ex ante*, by the relevant unit.

This also indicates that, even if units know B_g in advance, they may not be able to suspect whether a type I or II error is made in the evaluation of their eligibility to the benefit. Concern about the true level of entitlement B^* -- as opposed to concern about B_g only -- is directly relevant solely to the government agency and to the take-up analyst (and to social policy makers).

In our notation, assumption (a) says that $\epsilon_g \equiv \epsilon_p$ and, furthermore, that there is no uncertainty about B_p^b . Because of this ϵ_p and ϵ_a are now observationally equivalent, and we can define and work with ϵ , where $\epsilon = \epsilon_p - \epsilon_a (= \epsilon_g - \epsilon_a)$. We call ϵ a "modelling error" or "entitlement discrepancy", that is, it embodies the difference between entitlement as assessed by the government agency and that as appraised by the analyst: $\epsilon = B_g - B_a$. We also note that in these circumstances $B^* + \epsilon_p = B^* + \epsilon_g = B_a - \epsilon_a + \epsilon_g = B_a + \epsilon$.

This assumption also automatically removes the second hurdle of the claiming procedure, since anyone choosing to seek a benefit will do so only if he knows that $B_g > 0$. That is, all those finding it reasonably profitable to make a benefit application will be granted eligibility by the government agency: whenever $D=1$, we have $A=1$. Since, in reality, not everyone who requests a state benefit is conceded eligibility, making this assumption will tend to attribute to high costs and low perceived entitlement B_p too much of the weight for not receiving a state benefit. Thus, *ceteris paribus*, our estimates may tend to undervalue the perceived level of entitlement and overvalue the real costs of having made a request for successful claimants.

Not allowing for application refusals may not, however, overestimate the level of aggregate costs **incurred** in the benefit application process since in

estimating such aggregate deadweight costs the analyst also ought to account for the costs incurred by those who make a request but are judged ineligible. In other words, the aggregate inconvenience caused by the claiming process must be the sum of the inconvenience experienced both by those who have requested the benefit successfully and by those who have been refused it. Our "entitlement certainty" assumption may overestimate the former inconvenience but it neglects the latter, and we thus cannot tell here whether it leads to an important bias in the estimation of the burden created by the benefit application process.

Assumption (a) has at least one other effect, which tends to counteract the bias just outlined. In the absence of overly strong risk aversion, and other things being the same, feeling uncertain about the value of B_g tends to yield an expected monetary benefit to claiming that is larger than the benefit one would enjoy from the certainty of the mean value of entitlement. This is because if negative values of B_g were to arise in the application process a unit would only derive a zero, non-negative, benefit B_g^b (thus acting analogously to an insurance contract), but there are no such limits on the upper value of the benefit B_g^b payable for positive entitlements. This therefore yields an expected benefit to claiming larger than that which would be obtained from using with certainty the expected level of entitlement. Because, in our applied econometric analysis, we neglect uncertainty of entitlement on the part of the units and therefore underestimate the corresponding expected benefit to seeking a state benefit, our parameter estimates will, *ceteris paribus*, tend to overestimate the average B_g^b and underestimate the true cost of requesting a state benefit.

(b) Cost Certainty

E_p is known *ex ante* to the units but remains unobservable to the analyst. Thus, units are presumed to know well beforehand the inconvenience they would incur from seeking the state's support. Because of assumption (b), E_a and E_p become observationally equivalent and we may define E such that $E=E_a+E_p$. The combination of assumptions (a) and (b) removes all uncertainty in the choice of units to request or not the state benefit. It does not, however, alleviate the imperfection of information which the analyst has on the characteristics of the units and on the agency's rating of entitlement.

Assumptions (a) and (b) considerably reduce the burden of evaluating the probability that a unit is observed to receive a benefit. Assessing this probability now reduces to cumulating the density of ϵ and E along the range of ϵ and E over which a unit finds that the certain utility from requesting the benefit exceeds that of not requesting it. The conditional probability of observing a receipt, given B_g ⁷ and E , becomes:

$$P(D=1 \cap A=1 | B_g, E) = P(D=1 | B_g, E) = \begin{cases} 1, & \text{if } U(N^0 + B_g^b, Q' - E) > U(N^0, Q^0) \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

with:

⁷ We switch to the conditional probability given B_g instead of B^* since the observational equivalence of ϵ_a and ϵ_g renders it impossible to say anything on B^* -- unless we are to make the special assumptions that either ϵ_a or ϵ_g is equal to zero, as we shall do for some illustrative purposes later.

$$B_g^b (= B_p^b) = \begin{cases} 0, & \text{if } B_g (= B_a + e) \leq 0 \\ B_g, & \text{if } B_g (= B_a + e) > 0 \end{cases} \quad (13)$$

(c) Linear Utility Function

We finally work with a linear approximation of the general utility function $U(N, Q)$. Although not essential, this approximation is computationally useful, and the adoption of more elaborate specifications of U could quickly render the computational process impracticable. Calling U_N and U_Q the first-order derivatives of U with respect to N and Q , choosing to request the state benefit given B_g and E then reduces to verifying that:

$$U_N(N^0 + B_g^b - N^0) + U_Q(Q' - E - Q^0) = U_N \cdot B_g^b + U_Q(Q' - E - Q^0) > 0 \quad (14)$$

or $B_g^b > [X\alpha + E]$, with $X\alpha = \frac{U_Q}{U_N} \cdot (Q^0 - Q')$

where we have redefined E as $U_Q/U_N \cdot E$. It is this inequality which will underlie much of our econometric analysis. In the square brackets is the monetary equivalent of the loss in utility -- or the informational, transactional, psychological or other costs, expressed in the same unit as B_g^b -- induced by the act of requesting the benefit. We denote the non-negative (monetary equivalent) costs to claiming by $X\alpha + E$, where X is a vector of income unit characteristics, α is a vector of parameters conformable with X , and E represents any unobservable individual cost components.

This linearisation of the utility function prevents, among other things, a structural estimation of risk aversion in the form of the utility function, although the presence of risk aversion will influence the level of reduced-form cost

parameters. That is, if the linearisation does not approximate well the true utility function, U_Q/U_N will vary over Q and N and this will then affect the estimated value of α .

We may now summarise the formalisation of the simplified model we shall adopt for our empirical investigation. The combination of our three assumptions reduces the computation of the probability of a receipt to:

$$P(\text{successful receipt observed}) = \int \int_{E \ \epsilon} (D | B_g, E) f(E, \epsilon) d\epsilon dE \quad (15)$$

$$\text{where } (D | B_g, E) = \begin{cases} 1, & \text{if } B_g^b > X\alpha + E \\ 0, & \text{otherwise} \end{cases}$$

Thus, the probability of a successful claim being registered equals the cumulative distribution over the ranges of ϵ and E of the event that the state benefit exceeds the claiming cost, when the cost to claiming varies with E and when B_g is $B_a + \epsilon$. In other words, $P(\text{successful receipt observed})$ is the probability that the net benefit to requesting a state benefit be non-negative, given that the analyst cannot observe with perfect accuracy the level of benefit to which a unit is entitled, and given that it is not possible either to use observable characteristics to measure with certainty the degree of inconvenience $X\alpha + E$ incurred in seeking state support.

Figure 3.2 can help further our understanding of the structure of the simplified model which we intend to develop and apply in this and the next chapter. On the horizontal axis we feature the size of the cost of claiming the state benefit, which we can assess up to a stochastic and unobservable term E . On the vertical axis, we find the size of the entitlement, which is determined up to a stochastic term ϵ ; above the horizontal axis, the value of the benefit is positive and

equals the value of entitlement, but below it no benefit is payable for the unit is then not deemed entitled to receive the particular state support. Area A of Figure 3.2 denotes the region of the cost and entitlement space where a unit will choose to claim: the unit is then positively entitled to the state's support and the size of the benefit exceeds the level of the costs to claiming it. Hence, the likelihood that a claim will be made will equal the likelihood that the unit finds itself in area A rather than in area B (where costs exceed the value of the benefit) or C (where the unit is not entitled):

$$P(\text{successful receipt}) = P(A)/[P(A)+P(B)+P(C)]$$

This is equivalent to the previous equation. Conversely, the probability that no benefit receipt will be recorded equals the sum of the probability that the unit does not find it worthwhile to make a benefit application (region B) or that it is not deemed to be eligible (area C). Thus,

$$P(\text{no successful receipt}) = [P(B)+P(C)]/[P(A)+P(B)+P(C)]$$

(d) The Simplified Model and the Previous Studies

The derivation of the above model has demanded considerable analytical simplifications, but these have to be seen in the context of the much stronger restrictions implicit in the preceding empirical work. It is for this reason that we pause for a moment to reexamine briefly some of the analytical features of previous inquiries on the take-up of state benefits, particularly for those British studies reviewed above.

In all these studies, those not **calculated** or **modelled** to be eligible to the benefit are simply omitted from the investigation. Ignoring these observations in our econometric model would, in general, thwart a proper incorporation of

entitlement as a determinant of the net benefit to taking up and, in particular, make it difficult to incorporate soundly the presence of errors of modelling, both random and systematic. This is because deleting from our sample all observations for which (calculated) entitlement is non-positive would censor those units for whom entitlement is more likely to have been overestimated, potentially generating well-known biases in the resulting parameter estimates of the distribution of ϵ . As importantly, this sample truncation plausibly generates biased evidence on the probabilities of claiming a state benefit. Removing from the sample those not calculated to be eligible to receive state support would -- in the presence of purely random modelling errors ϵ^8 -- bias upwards the estimate of the costs involved in taking up **and** bias downwards the individual probability of claiming. Furthermore, even if no random errors of modelling were made in the process of calculating entitlement, biases on the cost estimates would of course still occur in the presence of systematic biases in measuring entitlement, or, for that matter, when a failure to account for features that affect the likelihood of observing a claim -- such as units' confusion of benefits -- can impart a spurious effect to costs characteristics.

2- Identifying Modelling Errors

The recognition of modelling errors is probably of importance in all areas of applied economic analysis, but it is particularly so in the use of micro survey data for which systematic as well as random measurement errors are suspected. A main issue is whether we can identify the empirical importance of such errors

⁸ That is, ϵ has mean zero.

without resorting to the use of information gathered in other surveys, surveys which incidentally may also suffer from similar defects. The analysis presented below suggests that we can, as long as there exists an assessable relationship between the economic variable subject to measurement errors and some other data (from the survey) for which such errors are either unlikely or are of negligible magnitude.

Take a variable y , sampled accurately, which is a function $f(z+\epsilon)$ of $z+\epsilon$, where ϵ is an error in measuring the "true" z . An obvious case in which ϵ is clearly identifiable is when f is both fully observed and bijective, viz, when there exists a known one-to-one relationship between $z+\epsilon$ and y . ϵ can then be found for every observation of y and z . Even if f is not bijective (e.g., y is a binary variable taking values of 0 and 1 for respective ranges of $z+\epsilon$ and therefore there does not exist a one-to-one relationship between $z+\epsilon$ and y) we will normally be able to identify at least some of the parameters of the distribution of ϵ . If the functional relationship between z and y is unknown or depends on unobserved parameters, then less information on the distribution of ϵ may generally be gathered.

y may, of course, also depend upon other observed factors X or additional random terms such as E , yielding the relationship:

$$y = f(z+\epsilon, X, E) \quad (16)$$

In some instances it may yet be possible to segregate the individual effects of $z+\epsilon$, X and E , and thus to identify at least some of the parameters of the separate distributions of ϵ and E .

The identification problem which will occupy us here relates the claims of

state benefits reported in a survey to an analyst's calculated distribution of entitlement. The two variables are clearly related and there is a suspicion that, on the one hand, entitlement data (based, for instance, on weekly household incomes) may exhibit various sampling inaccuracies, while, on the other hand, the report of whether an income unit has successfully claimed a state benefit would not be subject to substantial errors.⁹

As already seen, there is, however, a complication that turns out to yield additional benefits: the take-up of a state benefit also depends on the various inconveniences associated with the act of claiming. Some of these "costs" will depend on observed socio-economic characteristics X , but others will be strictly non-observable, that is, they may be subsumed under an additional random term E . The imperfectly (because of ϵ) observable value of entitlement determines whether a unit is at all entitled to a state benefit; if so entitled, the imperfectly observable value of claiming costs then resolves whether it is profitable for the unit to request the available benefit. We shall see even more clearly later that because ϵ and E have such crucially different effects on the likelihood of observing a particular value of y , we will be able to identify the relative parameters of their distribution.

C- An Application to the Take-up of Supplementary Benefits in

⁹ Note that we shall not consider the **amount** (although, for our empirical application, it happens to be available in our survey data) of the state benefit receipt, which would not necessarily be more accurate than our calculations of entitlement, but simply the (0,1) report of whether a state benefit was taken up. As we shall see later on, however, even this (0,1) variable may be subject to survey errors.

Britain

1- Derivation of the Model

Our computer model of the 1985 British tax and benefit¹⁰ system allows us to compute for each unit of a sample of Family Expenditure Survey (FES) income units a level of entitlement to various state benefits. We choose to apply our analysis to the claims of Supplementary Benefit (SB)¹¹. This is the programme most directed to the non-working poor. The size of the benefit depends on whether the recipients are deemed to be entitled to long-term (for people above the age of 59 or for those disabled in specified ways) or short-term requirements, and on the characteristics of the income unit. Additional requirements (and thus greater SB benefits) are also granted to those with housing requirements not covered by Housing Benefits and they strictly cover the mortgage costs incurred by the poorer owner-occupiers. The level of a unit's entitlement to SB then equals its level of requirements minus an assessed amount of resources or income, which, as the SB rules specify, is net of the basic pensions and unemployment benefits received and of the National Insurance Contributions and income taxes paid. Units with savings and capital in excess of £3000 or in full-time work cannot qualify. The benefit is payable for as long as the eligibility

¹⁰ See Chapter I.

¹¹ The Supplementary Benefits scheme was replaced in 1988 by a new Income Support programme (see Appendix B). The entitlement and payment rules were somewhat simplified, and standards common to the administration of Family Credit (which replaced Family Income Supplement) and Housing Benefits were established regarding capital and income limits. The calculation of rates (or "requirements" in 1985) was also eased. Notwithstanding these changes, the structure of Income Support remains broadly that of SB, and the results presented here are therefore still relevant to UK policy.

conditions are fulfilled and changes in circumstances must, in principle, be notified to the government agency.

Thus, B_a shall be the calculated entitlement to supplementary benefits calculated by our computer model of the 1985 British tax and benefit system. This assessed benefit entitlement will be negative for those income units, say, reporting relatively large non-labour and labour income. This degree of entitlement may not, however, correspond to that as appraised by the government agency (B_g) such that errors of various sorts will lead to a divergence ϵ between B_a and B_g :

$$B_g = B_a\beta_1 + \epsilon \quad (17)$$

with β_1 set to 1 later in the estimation process. Actual benefits payable B_g^b are as follows:

$$B_g^b = \begin{cases} 0, & \text{if } B_a\beta_1 + \epsilon = B_g \leq 0 \\ B_a\beta_1 + \epsilon, & \text{if } B_a\beta_1 + \epsilon = B_g > 0 \end{cases} \quad (18)$$

We follow the suggestion of Atkinson (1989) and model explicitly the interdependence that may exist between one's entitlements to various benefits. A recipient of SB would receive Housing Benefits (HB) at the level of a "certified claimant", while those not in receipt of SB would be entitled to a generally lower level of HB, *viz*, that of a "standard claimant". We call $BHCS$ the non-negative difference between these two HB levels. We may then integrate the impact of the decision to claim SB upon the level of HB payable. In estimating the parameters of our model we will construct an index β_2 of whether units, in choosing to take up SB, make their choice taking as unambivalent their accrued HB level. $\beta_2 BHCS$

will accordingly represent the perceived increased housing payment from which a potential SB recipient will benefit by claiming his positive supplementary benefit entitlement¹².

Keeping in mind that claiming a benefit also involves non-negative costs ($X\alpha + E$, which we may consider as being the average weekly costs of requesting and being in receipt of SB) we may now define "net benefit" NB as the supplementary benefit's net value to a unit thinking about requesting it:

$$Net\ benefit = NB = \begin{cases} -X\alpha - E, & \text{if } B_a \cdot \beta_1 + \epsilon < 0 \\ -X\alpha - E + B_a \cdot \beta_1 + \beta_2 BHCS + \epsilon, & \text{if } B_a \cdot \beta_1 + \epsilon \geq 0 \end{cases} \quad (19)$$

An income unit will claim the benefit if its net value, given in the above equation, is positive.

Hence, respectively grouping those income units observed to claim and not to claim into the sets C and NC , we note that the log-likelihood function $\ln L$ of the vector of independently¹³ sampled observed decisions D to claim or not to claim is given by

$$\ln L(D; parameters) = \sum_C \ln Pr(NB > 0) + \sum_{NC} \ln Pr(NB \leq 0) \quad (20)$$

To develop further this log-likelihood function we wish to specify the working distributions of ϵ and E and to derive that of NB . We assume that ϵ has

¹² We measure the value of $BHCS$ at the point at which entitlement to SB is calculated to end.

¹³ For an evaluation of this explicit "independence" assumption, see the discussion below.

a normal distribution f_ϵ with mean μ_ϵ and variance σ_ϵ^2 , and that E has a truncated (from below, at $-X\alpha$, such that costs $X\alpha+E$ are always non-negative) normal distribution f_E with untruncated zero mean and variance σ_E^2 . E and ϵ are assumed to be independently distributed. $f_{\epsilon,NB}$ is the joint p.d.f. of NB and ϵ : the distribution of NB is truncated upwards from $B_g^b + \beta_2 BHCS$, viz, NB can never exceed $B_g^b + \beta_2 BHCS$ for costs are non-negative.

We can integrate for each observation over the entire range of possible modelling errors ϵ and over the appropriate set of net benefits NB to yield the likelihood function of our sample:

$$\ln L = \sum_C \ln \int_{-\infty}^{\infty} \int_0^{\infty} f_{\epsilon,NB}(\epsilon, NB) dNB d\epsilon + \sum_{NC} \ln \int_{-\infty}^{\infty} \int_0^0 f_{\epsilon,NB}(\epsilon, NB) dNB d\epsilon \quad (21)$$

To obtain the explicit form of $f_{\epsilon,NB}(\epsilon, NB)$ we perform a transformation of the joint p.d.f. $f_{\epsilon,E}(\epsilon, E)$:¹⁴

$$f_{\epsilon,E}(\epsilon, E) = f_\epsilon(\epsilon) f_E(E) = \begin{cases} \frac{1}{\Phi\left(\frac{X\alpha}{\sigma_E}\right) \sigma_E \sigma_\epsilon} \phi\left(\frac{\epsilon - \mu_\epsilon}{\sigma_\epsilon}, \frac{E}{\sigma_E}; \rho_{\epsilon,E}=0\right), & E > -X\alpha \\ 0, & elsewhere \end{cases} \quad (22)$$

Hence, keeping in mind the definition of NB and through appropriate substitution, we find that the p.d.f. $f_{\epsilon,NB}(\epsilon, NB)$ is defined by

¹⁴ $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability and cumulative density functions of the standard normal distributions; $\phi(\cdot, \cdot; \rho)$ and $\Phi(\cdot, \cdot; \rho)$ are analogous for the standard bivariate normal distribution with correlation coefficient ρ .

$$f_{\epsilon, NB} = \frac{1}{\Phi\left(\frac{X\alpha}{\sigma_E}\right)} \begin{cases} \frac{1}{\sigma_\epsilon \sigma_E} \phi\left(\frac{\epsilon - \mu_\epsilon}{\sigma_\epsilon}\right) \phi\left(\frac{NB + X\alpha}{\sigma_E}\right), & \text{if } \epsilon < -B_a \beta_1, NB \leq 0 \\ 0, & \text{if } \epsilon < -B_a \beta_1, NB > 0 \\ \frac{1}{\sigma_\epsilon \sigma_{NB}} \phi\left(\frac{\epsilon - \mu_\epsilon}{\sigma_\epsilon}, \frac{NB + X\alpha - K}{\sigma_{NB}}; \rho_{\epsilon, NB}\right), & \text{if } \epsilon \geq -B_a \beta_1, NB \leq K + \epsilon - \mu_\epsilon \\ 0, & \text{if } \epsilon \geq -B_a \beta_1, NB > K + \epsilon - \mu_\epsilon \end{cases} \quad (23)$$

with

$$\begin{aligned} \sigma_{NB} &= \sqrt{\sigma_\epsilon^2 + \sigma_E^2} \\ K &= B_a \beta_1 + \mu_\epsilon + \beta_2 BHCS \\ \rho_{\epsilon, NB} &= \frac{\sigma_\epsilon}{\sigma_{NB}} \end{aligned} \quad (24)$$

It is possible to check that $f_{\epsilon, NB}(\epsilon, NB)$ is an appropriate p.d.f. by confirming that its integration over its whole domain equals 1. For values of $\epsilon < -B_a \beta_1$ SB are always zero and the net benefit to a request is both negative and independent of the error ϵ made in modelling the unit's entitlement: the bivariate distribution can then be expressed as a product of univariate normals. For values of ϵ above $-B_a \beta_1$ modelling errors ϵ do affect and are therefore correlated with net benefit NB , which can be no greater than $B_g^b + \beta_2 BHCS$.

After substituting (23) in (21), the log-likelihood function $\ln L$ becomes:

$$\ln L = \left\{ \begin{array}{ll} - \sum_{C,NC} \ln \Phi \left(\frac{X\alpha}{\sigma_\epsilon} \right) & (i) \\ + \sum_C \ln \int_{-B_a\beta_1}^{\infty} \int_0^{K+\epsilon-\mu_\epsilon} \frac{1}{\sigma_\epsilon \sigma_{NB}} \phi \left(\frac{\epsilon-\mu_\epsilon}{\sigma_\epsilon}, \frac{NB+X\alpha-K}{\sigma_{NB}} ; \rho_{\epsilon,NB} \right) dNB d\epsilon & (ii) \\ + \sum_{NC} \left[\ln \int_{-\infty}^{-B\beta_1} \int_{-\infty}^0 \frac{1}{\sigma_\epsilon \sigma_E} \phi \left(\frac{\epsilon-\mu_\epsilon}{\sigma_\epsilon} \right) \phi \left(\frac{NB+X\alpha}{\sigma_E} \right) dNB d\epsilon & (iii) \right. \\ \left. + \int_{-B_a\beta_1}^{\infty} \int_{-\infty}^0 \frac{1}{\sigma_\epsilon \sigma_{NB}} \phi \left(\frac{\epsilon-\mu_\epsilon}{\sigma_\epsilon}, \frac{NB+X\alpha-K}{\sigma_{NB}} ; \rho_{\epsilon,NB} \right) dNB d\epsilon \right] & (iv) \end{array} \right. \quad (25)$$

The areas of integration corresponding to the (lower case) roman numbers are shown on Figure 3.3. On the vertical axis is plotted the value of entitlement as assessed by the government agency: $B_a\beta_1+\epsilon=B_g$. This is equivalent to Figure 3.2, which we have already discussed, with the following differences:

- (1) on the horizontal axis we plot instead of $X\alpha+E$ the value of NB , which is the derived random variable that appears in the final likelihood function and whose appearance simplifies the interpretation and the analytical derivation of $\ln L$;
- (2) we include in Figure 3.3 the impact of $\beta_2 BHCS$, which adds to the likelihood that a unit will find it worthwhile to request SB when it finds itself entitled to it.

On the horizontal axis of Figure 3.3 we thus show the level of net benefit NB enjoyed from the receipt of SB. Since net benefit NB cannot exceed the level of benefit eligibility B_g , plus $BHCS \cdot \beta_2$ -- an allowance for the level of benefits dependent on eligibility to SB -- we would not observe units in the shadowed area and the terms in (i) account for this truncation. The (ii) summation sign encompasses the truncated log-likelihood that one with a non-negative entitlement

B_g to SB will take up the benefit despite the presence of burdens to claiming. The area of (ii) is limited on the left by the vertical line above $NB=0$ -- at which point a unit would be indifferent between claiming and not claiming a benefit whose value B_g would be exactly cancelled by the burden of requesting it. (ii) is also limited on the right by the maximum value of NB which may flow from a grant of B_g , that is, by $B_g + BHCS \cdot \beta_2$. The (iii) and (iv) terms account for the likelihood of not choosing to request the state benefit. (iii) shows the area in which a non-positive agency entitlement B_g necessarily leads to a non-positive net benefit to a SB request; (iv) indicates the region for which a positive agency entitlement is nevertheless outweighed by costs to claiming and for which NB is thus negative.

The computational and analytical difficulty of $L_n L$ varies with its terms. The (i) elements are straightforward: they account for the truncation from below of the normal distribution of claiming costs, such that these be negative with probability zero. The elements (iii) lead to a direct integration of the respective univariate density functions and therefore cause no numerical optimisation problem: over the domain for which ϵ is smaller than $-B_a \beta_1$, errors of modelling ϵ and net benefits NB are indeed independent since B_g^b is then always zero. Calling $\Phi_{\epsilon, NB}$ the bivariate cumulative distribution function of ϵ and NB and omitting the correlation coefficient $\rho_{\epsilon, NB}$, we find that the (iv) elements, which account for the presence of a negative net benefit when a positive SB entitlement is observed, simplify to:

$$\begin{aligned}
& \int_{-B_a\beta_1}^{\infty} \int_{-\infty}^0 \frac{1}{\sigma_{\epsilon}\sigma_{NB}} \phi_{\epsilon,NB} \left(\frac{\epsilon - \mu_{\epsilon}}{\sigma_{\epsilon}}, \frac{NB + X\alpha - K}{\sigma_{NB}} \right) dNB d\epsilon \\
& = \Phi_{NB} \left(\frac{X\alpha - K}{\sigma_{NB}} \right) - \Phi_{\epsilon,NB} \left(\frac{-B_a\beta_1 - \mu_{\epsilon}}{\sigma_{\epsilon}}, \frac{X\alpha - K}{\sigma_{NB}} \right)
\end{aligned} \tag{26}$$

The most difficult to disentangle are the (ii) terms, of which (iv) is simply a special and easier case. Splitting the domain of integration, we find:

$$\begin{aligned}
\sum_C \left[\int_{-B_a\beta_1}^{\infty} \int_{-\infty}^{K+\epsilon-\mu_{\epsilon}} \frac{1}{\sigma_{\epsilon}\sigma_{NB}} \phi \left(\frac{\epsilon - \mu_{\epsilon}}{\sigma_{\epsilon}}, \frac{NB + X\alpha - K}{\sigma_{NB}} \right) dNB d\epsilon \right. \\
\left. - \int_{-B_a\beta_1}^{\infty} \int_{-\infty}^0 \frac{1}{\sigma_{\epsilon}\sigma_{NB}} \phi \left(\frac{\epsilon - \mu_{\epsilon}}{\sigma_{\epsilon}}, \frac{NB + X\alpha - K}{\sigma_{NB}} \right) dNB d\epsilon \right]
\end{aligned} \tag{27}$$

The last part of (27) then flows directly from the derivation of (26). After proceeding to a change of stochastic variables, viz, from (ϵ, NB) to $(\epsilon, NB - \epsilon)$, it occurs that:

$$\int_{-B_a\beta_1}^{\infty} \int_{-\infty}^{K+\epsilon-\mu_{\epsilon}} \frac{1}{\sigma_{\epsilon}\sigma_{NB}} \phi \left(\frac{\epsilon - \mu_{\epsilon}}{\sigma_{\epsilon}}, \frac{NB + X\alpha - K}{\sigma_{NB}} \right) dNB d\epsilon = \Phi \left(\frac{B_a\beta_1 + \mu_{\epsilon}}{\sigma_{\epsilon}} \right) \Phi \left(\frac{X\alpha}{\sigma_E} \right) \tag{28}$$

The log-likelihood function thus finally simplifies to:

$$\ln L = \begin{cases} -\sum_{C,NC} \ln \Phi\left(\frac{X\alpha}{\sigma_E}\right) & (i) \\ +\sum_C \ln \left[\Phi\left(\frac{B_a\beta_1 + \mu_\epsilon}{\sigma_\epsilon}\right) \Phi\left(\frac{X\alpha}{\sigma_E}\right) - \Phi\left(\frac{X\alpha - K}{\sigma_{NB}}\right) + \Phi_{\epsilon,NB}\left(\frac{-B_a\beta_1 - \mu_\epsilon}{\sigma_\epsilon}, \frac{X\alpha - K}{\sigma_{NB}}\right) \right] & (ii) \\ +\sum_{NC} \ln \left[\Phi\left(\frac{-B_a\beta_1 - \mu_\epsilon}{\sigma_\epsilon}\right) \Phi\left(\frac{X\alpha}{\sigma_E}\right) + \Phi\left(\frac{X\alpha - K}{\sigma_{NB}}\right) - \Phi_{\epsilon,NB}\left(\frac{-B_a\beta_1 - \mu_\epsilon}{\sigma_\epsilon}, \frac{X\alpha - K}{\sigma_{NB}}\right) \right] & (iii) \end{cases} \quad (29)$$

(iv)

It may be worthwhile to examine some of the interesting features exhibited by our econometric analysis. Firstly, the recognition of the link between benefits adds here to the likelihood that one will be observed to claim SB, since the grant of additional housing benefits and of passport benefits enlarges the area of integration for which NB is positive. In different circumstances, it might be that the receipt of a benefit is decreased by the claim of another; in that case the region of integration for the take-up of the second benefit would look like Figure 3.4, decreasing (given the value of the parameters) the likelihood of a unit taking it up.

Secondly, because NB is linear in costs ($X\alpha + E$) but, though linear in B_g^b , is nonlinear in ϵ , it is possible to have an estimate of the relative variance of costs and modelling errors. Furthermore, the availability of this estimate does not depend on the distributional assumptions made, nor is it a consequence of the adoption of a particular functional form. The identification of the relative distributional parameters of ϵ and E is an intrinsic feature of the sample distribution of claims and non-claims. Varying the variance of ϵ relative to that of E will fundamentally alter the distribution and likelihood functions of net benefits to claiming, as can be seen from a close look at the log-likelihood function (29). For instance, and more intuitively, the probability of observing an income unit

claim when it is calculated to be entitled will often be maximised by a relatively low variance of modelling errors -- the same applies to a unit observed not to claim when it is not thought to be allowed to receive a benefit. Conversely, the observation that a unit not thought to be entitled claims a benefit tends to suggest a variance of modelling errors large relative to that of claiming costs. The asymmetry can be summarised as follows: explaining the non-receipt of SB of a unit with positive calculated entitlement can be done through appealing either to errors of modelling or large costs (unobservable or not) to claiming, but accounting for the claim of a unit with a seemingly negative entitlement to the benefit can only imply the recognition of the presence of "errors of modelling".

Thirdly, using the calculated value of entitlements further allows the transformation of the estimate of the relative variances into estimates of the respective absolute variances. This additional result is obtained by bringing in the constraint that $\beta_1=1$. This also enables us to derive direct estimates of the costs involved in claiming state support, estimates that are consistent with the presence of entitlement inaccuracies.

2- The Numerical Optimisation¹⁵

The log-likelihood function derived above is a menacing one, and, since we cannot rely on standard statistical and econometric packages to perform the optimisation, its numerical maximisation can be difficult and resource-consuming. We therefore indicate in this section some of the methods used to find the values of the parameters α , β_1 , β_2 , σ_e , and σ_E that yield the maximum of $\ln L$ for a sample

¹⁵ This section owes much to the patient and kind advice of Joanna Gomulka.

of observations $\{C, NC\}$.

(a) Identification and Simplification of $\ln L$

We must first notice that, as $\ln L$ presently stands, none of the parameters of the likelihood function can be identified; to see this, we note that to multiply all parameters by a factor k would leave the value of $\ln L$ unchanged. An identifying constraint is thus required, and it is later provided by setting β_1 to one. For the numerical optimisation, however, it is more convenient to let β_1 vary and to set $\sigma_{NB}=1$, and subsequently to redefine $\sigma_e=\cos \xi$ and $\sigma_E=\sin \xi$ for simplicity. Moreover, we shall see that many of the results obtained do not require setting β_1 to one. Finding the estimated standard errors $\hat{\sigma}_e$ and $\hat{\sigma}_E$ then reduces to optimising over the domain of ξ , which ranges from 0 to $\pi/2$; the ratio of the standard error of costs over that of calculated entitlements is simply $\sin \xi/\cos \xi=\tan \xi$. We can then simplify greatly the numerical optimisation for $\ln L$ by making appropriate changes of variables:

$$\ln L = \begin{cases} -\sum_{C, NC} \ln \Phi(X\bar{\alpha}) & (i) \\ +\sum_C \ln [\Phi(B_a\bar{\beta}_1 + \bar{\mu}_e) \cdot \Phi(X\bar{\alpha}) - \Phi(\bar{K}) + \Phi_{e, NB}(-B_a\bar{\beta}_1 - \bar{\mu}_e, \bar{K}; \rho)] & (ii) \\ +\sum_{NC} \ln [\Phi(-B_a\bar{\beta}_1 - \bar{\mu}_e) \cdot \Phi(X\bar{\alpha}) + \Phi(\bar{K}) - \Phi_{e, NB}(-B_a\bar{\beta}_1 - \bar{\mu}_e, \bar{K}; \rho)] & (iii) \end{cases} \quad (30)$$

where

$$\begin{aligned} \bar{\alpha} &= \frac{\alpha}{\sin \xi} ; \bar{\beta}_1 = \frac{\beta_1}{\cos \xi} ; \bar{\mu}_e = \frac{\mu_e}{\cos \xi} ; \\ \bar{K} &= X\bar{\alpha} \sin \xi - B_a\bar{\beta}_1 \cos \xi - \bar{\mu}_e \cos \xi - \beta_2 BHCS ; \\ \rho &= \cos \xi \end{aligned} \quad (31)$$

We therefore maximise $\ln L$ with respect to the new parameters $\bar{\alpha}$, $\bar{\beta}$, $\bar{\mu}$, β_2 and ξ , making the optimisation process less costly and much more likely to converge. Once we have found the estimates of these transformed parameters, we can then substitute them and obtain the estimated values of the underlying parameters of our distribution.

(b) Assessing $\ln L$ and its Gradient

We make use of the Newton algorithm E04KDF of the NAG Fortran library, for which we provide the vector of $\ln L$'s analytical gradient. The derivative of a univariate cumulative distribution function (c.d.f.) is the relevant univariate probability density function (p.d.f.), while the derivative of a bivariate c.d.f. $\Phi(r_1, r_2; \rho)$ with respect to one of its random variables r_1 or r_2 is the product of a conditional c.d.f. and a univariate p.d.f., e.g.,

$$\frac{\delta \Phi(r_1, r_2; \rho)}{\delta r_1} = \Phi(r_2 | r_1) \cdot \phi(r_1) \quad (32)$$

This product is easily computed. When a gradient involves the derivative of a (normal) bivariate c.d.f. with respect to its correlation coefficient, we make use of the fact that

$$\frac{\delta \Phi(r_1, r_2; \rho)}{\delta \rho} = \phi(r_1, r_2; \rho) \quad (33)$$

We also need to compute $\ln L$ itself and thus the values of the normal bivariate c.d.f. $\Phi_{\epsilon, NB}$; no NAG subroutines are directly available for this purpose, and we thus compile two routines. The first one cheaply yields results accurate

to about four decimals over a limited range of the bivariate distribution; the second makes use of numerical integration to generate results to any desired level of accuracy over the whole range of r_1 , r_2 and ρ .

Because we have chosen not to discard any of the relevant sample observations, it is not entirely surprising that the optimisation process will need to cope with some "extreme" values; for instance, there are in our sample a few income units with very low negative (large in absolute value) calculated entitlements which still declare that they are taking up SB. Other examples occur when $\rho_{e,NB}$ tends to zero, when calculated entitlement is large for some observations for which no SB receipt is recorded, or when costs are estimated to be particularly low or large. The numerical value of the gradient of such extreme observations may be difficult to evaluate on a computer since it is often derived from a ratio of small numbers; that is, if L_i is small, so may $\delta L_i / \delta(\cdot)$ be, and $\delta(\ln L_i) / \delta(\cdot) = (\delta L_i / \delta(\cdot)) / L_i$ will then be difficult to calculate accurately on a computer. We will then resort to using asymptotic gradient values, which is analogous to the use of Mills' ratio for the univariate ratio $\phi(r)/\Phi(r) \sim r$ when r is small.

To find these asymptotic bivariate ratios we apply l'Hôpital's rule, taking the derivative of the numerator $\delta L_i / \delta(\cdot)$ and of the denominator L_i with respect to the variable (cost, entitlement or $\rho_{e,NB}$) whose value is causing $\delta L_i / \delta(\cdot)$ and L_i to be small, and making appropriate simplifications and approximations. We use these asymptotic ratios whenever in the optimisation procedures $\ln L_i$ happens to be no greater than 10^{-10} . Similarly, the value of $\ln L_i$ must be approximated to its asymptotic level when costs to claiming $X\alpha$ are estimated to be especially low.

Conclusion

We have seen in this chapter how to justify and conduct the analysis of benefit take-up in the presence of divergences and errors in modelling entitlement, made both by the take-up analyst and by the administrative agency responsible for the allocation of state benefits. Our methodology, which may be extended to analogous microeconomic applications, makes use of the existence of an assessable economic relationship between data subject to random and systematic errors and other data for which such errors are a priori thought to be unlikely. The presence of an additional source of unobservable random errors does not prevent the relative parametric identification of the relevant distributions since the two sources of randomness commonly generate unambiguously distinguishable effects. We also noted that an application of our methodology to the take-up of Supplementary Benefits in Britain would yield absolute estimates of the distributions of modelling errors and of unobservable costs, as well as direct estimates of the monetary equivalents to the burden of claiming. It is to such estimation that we turn in the next chapter.

Figure 3.1

Eligibility and the Take-up of State Benefits

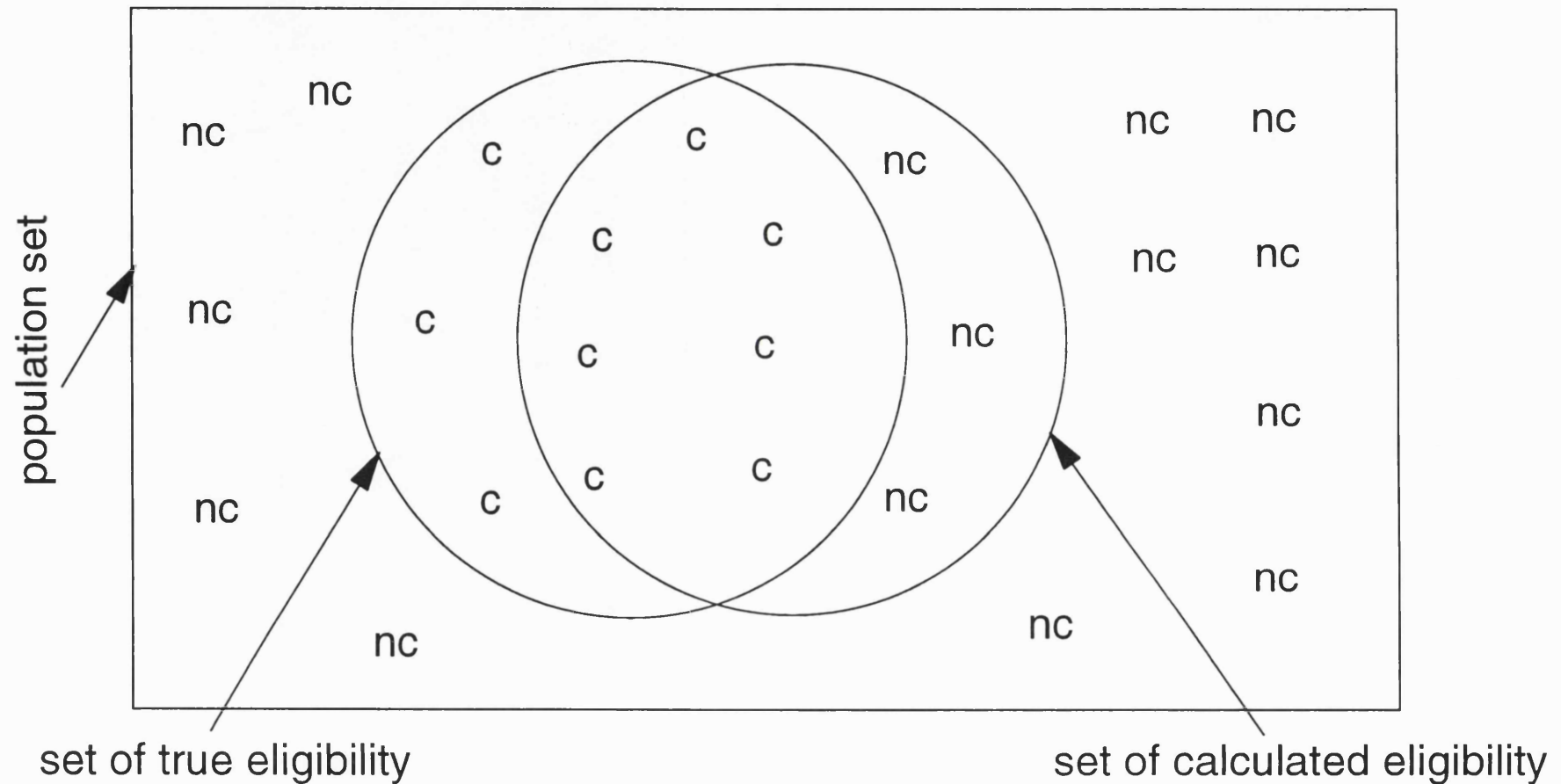


Figure 3.2

Claiming and not Claiming

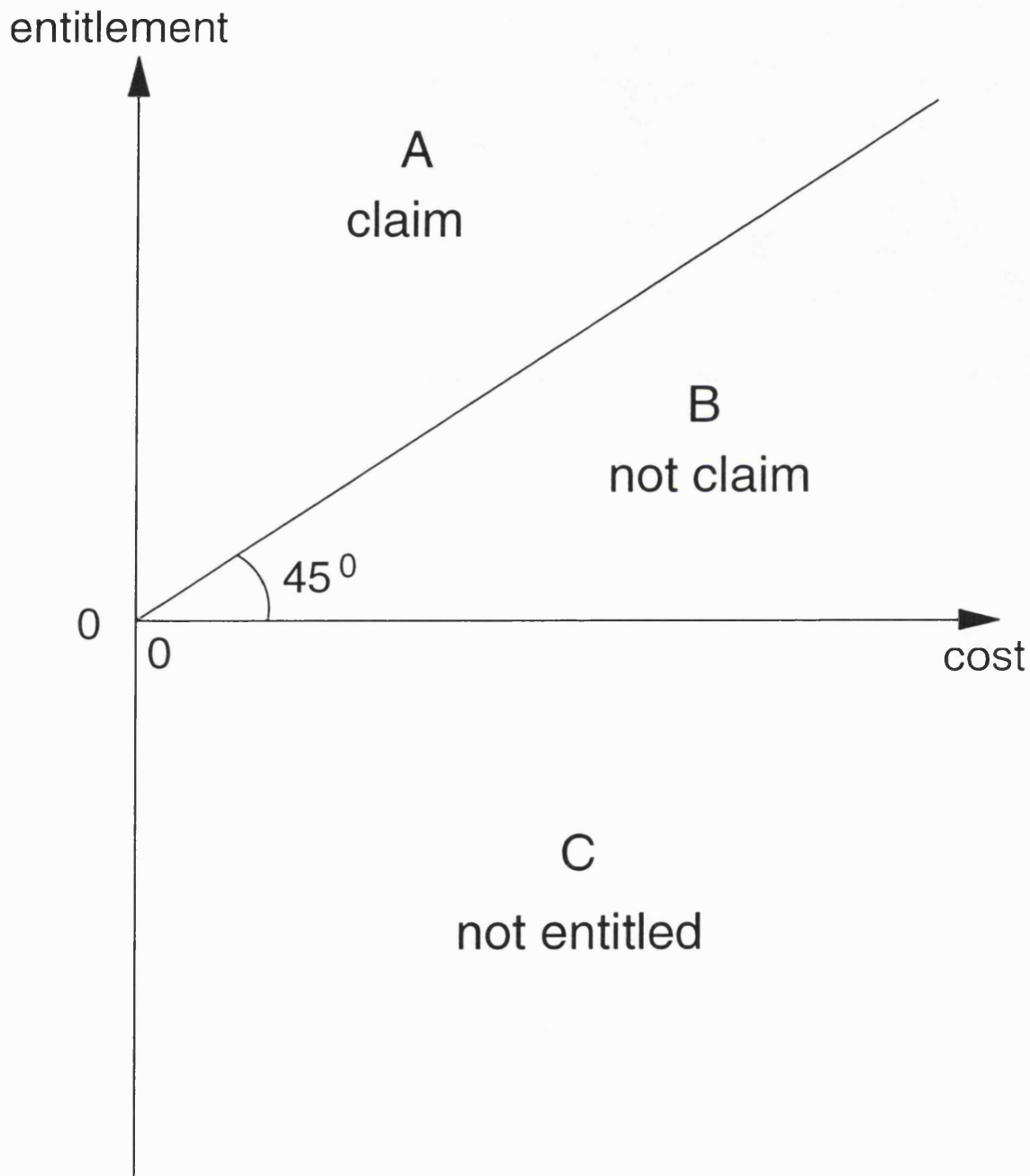


Figure 3.3

Entitlement and Net Benefit

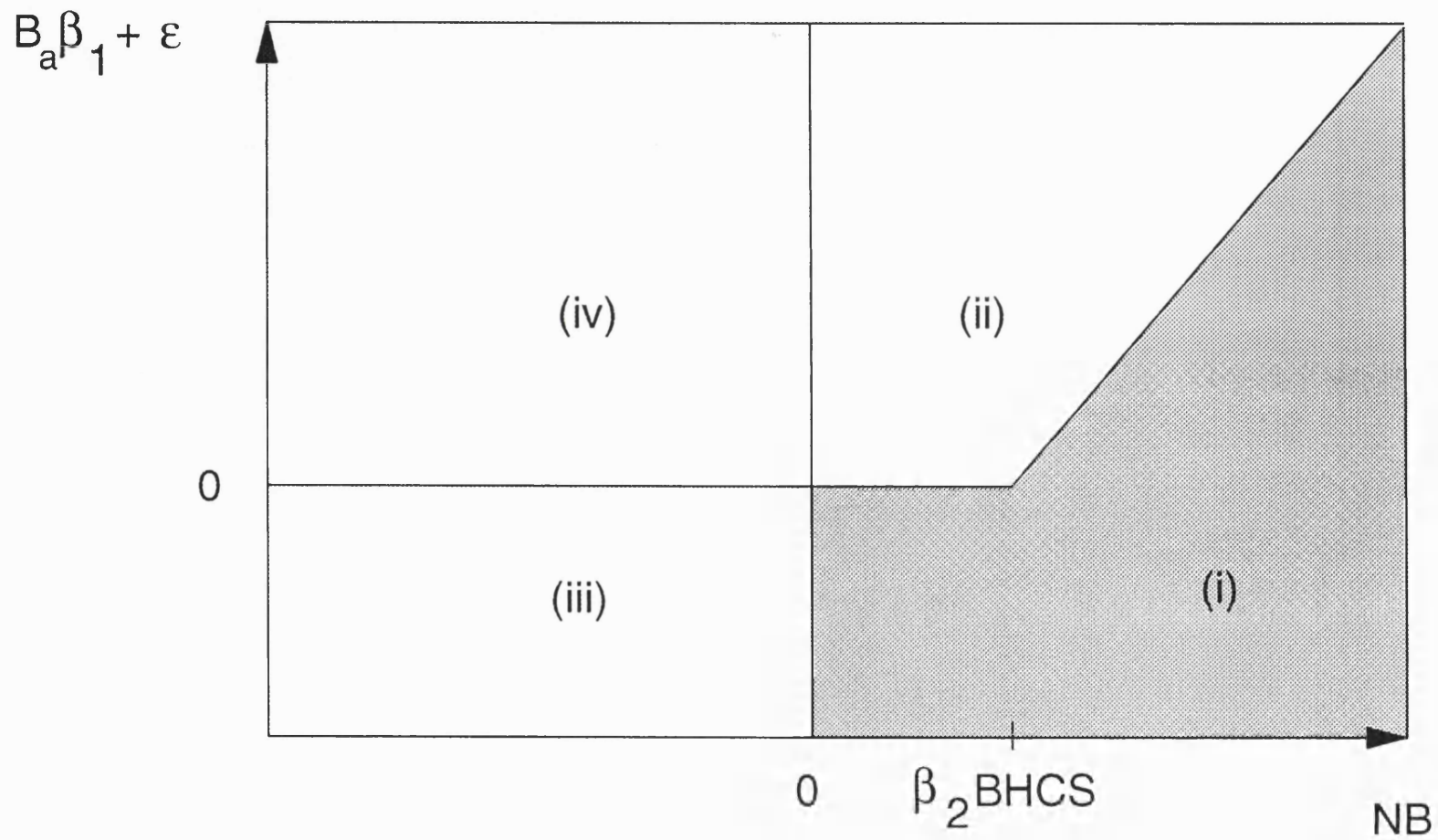
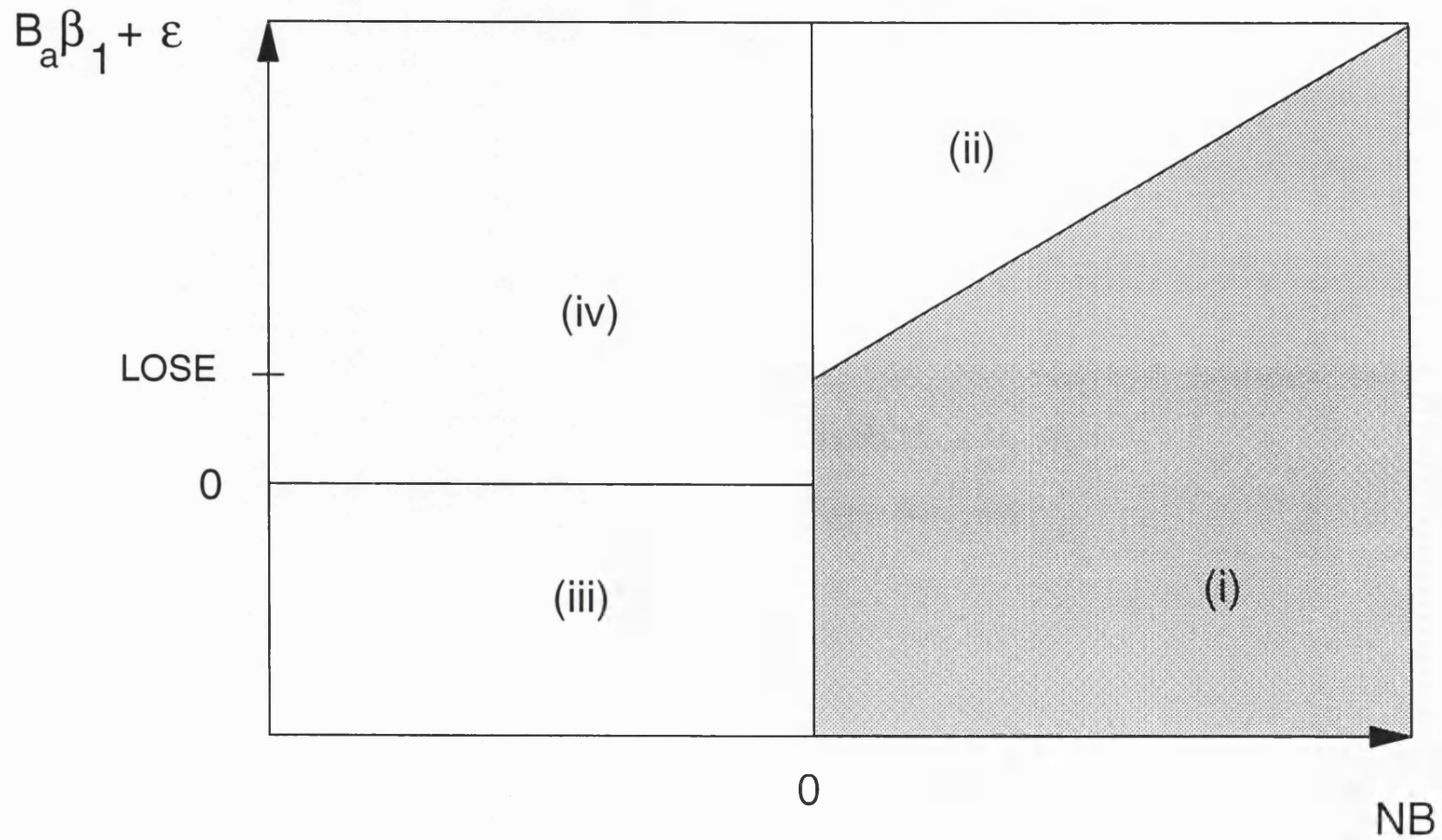


Figure 3.4

Entitlement and Net Benefit



Chapter IV: On the Determinants of the Take-up of Supplementary Benefits in 1985 Britain

A- Modelling of Entitlement Divergences and Some Survey Deficiencies

1- On Divergences in the Computation of Entitlements

We have already extensively discussed in chapter III the role and the econometric characterisation of "modelling errors" in understanding the take-up of state benefits. Administrative, data and computing imperfections will lead to random and systematic divergences in the computation of entitlement which may well be important for normative and positive purposes. In our applied work we will consider three classes of suspected systematic deficiencies, all three prompted by our use of the FES data and by the analysis that documents the expected accuracy of the survey's information. The first two speculated flaws will simply help shape the specification of μ_v but the third will demand a deeper restructuring of our econometric analysis:

- (1) Systematic differences in entitlement calculations by the government agency and by the analyst, stemming, for example, from general misreporting of income data and characteristics of units;
- (2) Systematic differences in entitlement calculations by the government agency and by the analyst, for specific groups;
- (3) Failure of pensioners to report accurately their receipt of supplementary benefits.

We will analyse the third (survey) deficiency in the next subsection. We intend to account for the first two discrepancies by providing a simple specification of μ_ε :

$$\mu_\varepsilon = RB \cdot \beta_3 \quad (1)$$

where RB is a vector of entitlement indicators and β_3 is a conformable vector of parameters showing the importance of RB in explaining the structural discrepancy ε between B_g and B_a . It is worthwhile remembering that "modelling errors" always refer here to the gap between the analyst's calculation of entitlement and the norm set by the government institution responsible for the administration of the benefit -- an institution which itself may not impute entitlement accurately and may not have correct information on the unit's income and other characteristics either. Hence, were units to conceal successfully part of their income both to the survey and to the administrators of the benefit, our entitlement indicator μ_ε would fail to pick up any entitlement "biases". We also note that parameters β_3 will generally be separately identifiable even if they correspond to variables in RB that are also found in the vector of cost characteristics X . The identification of such parameters is possible for the same reasons that the standard deviations of E and ε are also separately identifiable.

We shall allow for systematic and general divergences ε in the computation of entitlements by including a unit vector into RB . For discrepancies in the computation of the entitlement value of the members of specific groups, appropriate binary (dummy) variables may be provided. In particular, there is a suspicion that self-employment income is particularly prone to under-reporting

in the FES¹: we rely on the analytical simplicity of the procedure on which the award of SB is based to take explicit account of this plausible survey deficiency in our econometric analysis. We describe the method chosen in the following paragraphs.

The government agency's assessment of self-employed's entitlement to SB is

$$B_g = NEEDS - YGROS_g + \eta_g \quad (2)$$

where *NEEDS* are assessed by the government agency using the SB rules, η_g is a random error term, and $YGROS_g$ is the level of pre-SB income which the self-employed would reasonably declare (though not necessarily his real income) to the Department of Social Security (DSS) staff. B_g is thus (in the context of this study) the level of entitlement which we would ideally like to use in our econometric investigation to assess the level of costs incurred in the process of claiming SB. It may be, however, that for several reasons the information on self-employment income in the survey data leads to the computation of a different pre-SB income, viz, $YGROS_a$, with:

$$YGROS_g = b \cdot YGROS_a + a - \eta_a \quad (3)$$

If $b \neq 1$ or if $a \neq 0$, then $YGROS_g$ and $YGROS_a$ will generally differ, leading to systematic discrepancies between B_g and B_a . In particular, for $a > 0$ and $b > 1$, pre-SB

¹ On the reliability of FES data, see for instance Kemsley et al. (1980), Atkinson and Micklewright (1983), and Atkinson, Micklewright and Stern (1988).

income will be relatively and systematically underreported in the survey data: a allows for a fixed effect and b for underreporting that is proportional to the level of self-employment income. Combining the last two equations, we find that an appropriate or revised modelling of entitlement would take the form:

$$B_g \begin{cases} = NEEDS - b \cdot YGROS_a - a + \eta_g + \eta_a \\ = NEEDS - YGROS_a + (1-b) \cdot YGROS_a - a + \eta_g + \eta_a \\ = B_a + (1-b) \cdot YGROS_a - a + \eta_g + \eta_a \end{cases} \quad (4)$$

As before, B_a is the value of entitlement calculated using the survey information available. To comply with the above formulation, the vector of entitlement indicators RB will thus need to include $YGROS_a$ and a dummy variable for self-employment.

2- On Pensioners

"The number of pensioners reporting receipt of supplementary benefit in FES falls short of the number shown in the department's statistical enquiries by about 500 thousand, about a third of the total. The shortfall probably results from pensioners reporting their supplementary benefit as part of their retirement pension." [DSS(1989), p.6] This, as we will see, does not prevent the DSS from attempting to assess the rate of take-up for pensioners in 1985, although their method is rather crude. Instead of estimating what the underlying distributions of entitlements and claims are (as we shall attempt to do here), the DSS uses only the FES set of **declared** receipts of SB and then endeavours to determine the size of the limited set of eligible recipients that have not mistakenly declared receipt of a pension instead of SB payment.

This approach may well be fine as a ready way of finding a rough estimate

of take-up rates, but it is of no benefit to our econometric analysis. Alternatively, we might want simply to forsake the subsample of pensioners, to provide a separate analysis [e.g., Fry and Stark (1987)], or to make ad hoc adjustments for pensioners; these options, however, would not provide suitable and unbiased estimates of the structural parameters of the cost and net benefit distributions for the subsample of pensioners. This is because ignoring the likelihood of benefit confusion among pensioners would, everything else being the same, misleadingly swell the estimate of the apparent trouble to claiming and likewise suggest that the calculation of their entitlement B_a was biased upwards. Allowing for an explicit modelling of pensioner behaviour may enable us to reconcile the underlying distribution of take-up with that recorded in the survey, and may make it possible to highlight some pertinent consequences.

As for all other income units, a pensioner unit will choose to claim a state benefit if the net benefit NB to doing so is non-negative; that is, if D represents the decision to claim ($D=1$) and not to claim ($D=0$), we have:

$$\begin{aligned} NB > 0 &\Leftrightarrow D = 1 \\ NB \leq 0 &\Leftrightarrow D = 0 \end{aligned} \tag{5}$$

We are nonetheless restricted to observing the event D^* of whether the income unit declares its receipt of SB. For many groups of our sample units, we implicitly assume D and D^* are equivalent in likelihood: a unit taking up SB will reveal this in the survey, and if the unit does declare SB receipt, it is because it is receiving the benefit. For pensioners, however, we allow for the probability that a SB payment can be confused by a SB recipient with the receipt of a state

retirement pension. In what follows we neglect all other possible types of benefit confusion and concealment among our sample units. We represent the behaviour of pensioners as follows:

$$\begin{aligned} \text{if } D=1 \text{ (or } NB>0) \text{ then } & \begin{cases} D^*=1 \text{ with probability } \bar{P} \\ D^*=0 \text{ with probability } 1-\bar{P} \end{cases} \\ \text{if } D=0 \text{ (or } NB\leq 0) \text{ then } & D^*=0 \text{ with probability } 1 \end{aligned} \quad (6)$$

That is, pensioners who take up SB will correctly reveal it to the surveyors with probability \bar{P} , but mistakenly confound receipt of SB with that of another benefit (i.e., retirement pension) with probability $1-\bar{P}$. We define \bar{P} as $\bar{P}=1-BPROB\cdot\psi$. For those units who declare no receipt of retirement pensions **and** for those pensioners for whom it is possible to be reasonably certain that a benefit confusion has not occurred², we define $BPROB=0$ and thus assume that $\bar{P}=1$. For all other pensioners, $BPROB=1$ and there then remains to find the estimate of ψ . The likelihood of observing a recipient of retirement pension declare ($D^*=1$) or not declare ($D^*=0$) a claim of SB is thus:

² SB is allocated to the income unit as a whole and not to individual unit members, whereas extra retirement pension payments can be paid for dependant children or for a spouse and can be declared as such in the survey. However imprecise, this feature can make it possible to identify pensioner units not likely to have confused a SB receipt for a National Insurance retirement pension one. To ascertain the presence of such units, we use (if applicable) the declared receipt of a spouse's National Insurance pension, model it as a basic pension increase for a spouse, and find the corresponding minimum level of the total receipt of National Insurance retirement pension. If this minimum equals or exceeds the declared receipt of National Insurance retirement pension, we specify $BPROB=0$.

$$\begin{aligned}
L(D^*=1) &= \bar{P} \cdot P(NB>0) \\
L(D^*=0) &= (1-\bar{P}) \cdot P(NB>0) + P(NB \leq 0)
\end{aligned}
\tag{7}$$

There is, however, an additional difficulty in identifying the correct distribution of pensioners' entitlement and net benefit to claiming SB. If a pensioner unit mistakenly reports in the survey a receipt of SB, our calculation of its SB entitlement will be biased downwards since we will have entered into such the calculation of entitlement a misreported amount of (retirement) pre-SB income. That is, because retirement pensions enter the gross income base on which SB is awarded, the confusion of receipts of retirement pension and SB will not only affect the probability of observing a claim given that an SB award is received but it will also hamper our ability to calculate SB entitlements from the sample data.

Fortunately, however, we can calculate the entitlement to SB which would be computed if some of the amount of retirement pension declared in the FES was in fact a hidden SB receipt. We call this new calculated entitlement $B2_a$ (to which also corresponds the vector of entitlement indicators $RB2$). The difference between B_a and $B2_a$ is thus that B_a is calculated on the assumption that pensioners have not misreported SB as retirement pension, and that $B2_a$ is computed on the assumption that at least some of the declared receipt of retirement pension is not genuine. The amount of the possibly incorrect declaration of National Insurance retirement pension is determined by the difference between a derived minimum level of received National Insurance retirement pension and the declared receipt

of National Insurance retirement pension³. $B2_a$ is then found by attributing to the unit that minimum level of National Insurance pension, not the declared one.

Let's call T the ability of a pensioner unit to divulge correctly ($T=1$, else $T=0$) its receipt of SB when it does take it up. Then, whenever a pensioner receives SB,

$$T = \begin{cases} 1, & \text{with probability } \bar{P} \\ 0, & \text{with probability } (1-\bar{P}) \end{cases} \quad (8)$$

Hence, the disclosure D^* of a pensioner's SB claim is found as:

$$D^* = \begin{cases} 0, & \text{if } NB \leq 0, \text{ or if } NB > 0 \text{ and } T=0 \\ 1, & \text{if } NB > 0 \text{ and } T=1 \end{cases} \quad (9)$$

The likelihood of observing $D^*=1$ is as before:

$$L(D^*=1) = P(NB > 0, T=1) = \bar{P} \cdot P(NB > 0) \quad (10)$$

where we assume that the probability of claiming SB is independent of that of correctly revealing a claim. Similarly, the likelihood of observing $D^*=0$ (no claim reported) equals:

$$\begin{aligned} L(D^*=0) &= P(NB \leq 0) + P(NB > 0, T=0) = P(NB \leq 0) + P(NB > 0 | T=0) \cdot P(T=0) \\ &= P(NB \leq 0) + P(NB > 0) \cdot (1-\bar{P}) \end{aligned} \quad (11)$$

$NB2$ is the net benefit corresponding to the use of the measure of entitlement $B2_a$

³ See the last footnote for more details. The imputation of the levels of National Insurance retirement pensions from the use of the FES is discussed in Appendix A.

and $RB2$.

If it does appear that there exists a positive probability of misreporting the grant of a SB payment, we ought not to use B_a and RB alone as satisfactory measures of calculated entitlement over which to assess, say, expected claims or the rate of benefit take-up. Instead, we must then appraise the distribution of entitlement as a function of both (B_a, RB) and $(B2_a, RB2)$ since we are then uncertain about what we defined to be our own (viz, the analyst's) calculation of entitlement ought to be. Figure 4.1 might be helpful in seeing how we may do this. Its top part shows the way true take-up for pensioners is reflected into disclosures of SB claims in the survey. Our econometric analysis will enable both the estimation of the parameters describing the true taking up D of SB and the estimation of \bar{P} , thus making it possible to explain and recreate the distribution of data for pensioners. That is, once we find the estimated values of the parameters $\beta_1, \beta_2, \beta_3, \alpha, \sigma_E, \sigma_\varepsilon$ and the value of ψ in $1-BPROB \bullet \psi$, we will then be in a position to predict both the true receipts of SB and the expected declarations of such receipts in the FES data. The weight we shall then attach to (B_a, RB) and $(B2_a, RB2)$ as indicators of the correct entitlement figure for pensioners will depend on a unit's value of D^* , as is suggested by the bottom part of Figure 4.1. If $D^*=1$, viz, if a unit reveals its claim of SB, (B_a, RB) is the correct entitlement vector with probability 1: $P(RB \text{ is expected entitlement} \mid D^*=1)=1.0$. For $D^*=0$, that is, for those who do not report a SB receipt and for whom there is therefore a suspicion that a benefit confusion might have occurred, we ought to construct a more elaborate conditional distribution of entitlement. What is desired are the respective probabilities that (B_a, RB) or $(B2_a, RB2)$ be the correct expected entitlement figures given that we

know that the unit alleges not to claim SB. We note first that:

$$\begin{aligned} L((B_a, RB) \text{ is correct entitlement vector and } D^*=0) &= P(NB \leq 0) \\ L((B2_a, RB2) \text{ is correct entitlement vector and } D^*=0) &= (1 - \bar{P}) \cdot P(NB2 > 0) \end{aligned} \quad (12)$$

Then, since $P(r_1 | r_2) = P(r_1, r_2) / P(r_2)$, we see that given that we observe $D^*=0$ we expect (B_a, RB) and $(B2_a, RB2)$ with the following probability, as shown on the bottom of Figure 4.1:

$$\begin{aligned} P(RB \text{ is expected entitlement} \mid D^*=0) &= \frac{P(NB < 0)}{L(D^*=0)} \\ P(RB2 \text{ is expected entitlement} \mid D^*=0) &= \frac{(1 - \bar{P}) \cdot P(NB2 \geq 0)}{L(D^*=0)} \end{aligned} \quad (13)$$

Therefore, whenever our calculations involve pensioners who do not declare a receipt of SB, we shall account⁴ for the uncertain value of calculated entitlement and we shall use the distribution of (B_a, RB) and $(B2_a, RB2)$ with the probability distribution specified above. For instance, let's note that our calculated entitlement of a pensioner unit with $D^*=0$ is expected to be

$$\frac{P(NB \leq 0) \cdot B_a + (1 - \bar{P}) \cdot P(NB2 > 0) \cdot B2_a}{L(D^*=0)} \quad (14)$$

whereas its ex ante probability of claiming equals

⁴ Unless we indicate specifically that we do not do so.

$$\frac{P(NB \leq 0) \cdot P(NB > 0) + (1 - \bar{P}) \cdot P(NB2 > 0) \cdot P(NB2 > 0)}{L(D^* = 0)} \quad (15)$$

These reduce to B_a and $P(NB > 0)$ for those pensioners for whom $BPROB = 0$.

We are now able to proceed to the maximisation of our likelihood function, with the enhancements just made for pensioners. The final likelihood function is:

$$\begin{aligned} \ln L(D^*; \beta_1, \beta_2, \beta_3, \alpha, \sigma_E, \sigma_e, \psi \mid X, B_a, RB, B2_a, RB2, BPROB) = \\ \sum_{D^*=1} \ln[(1 - BPROB \cdot \psi) \cdot P(NB > 0)] + \sum_{D^*=0} \ln[(BPROB \cdot \psi) \cdot P(NB2 > 0) + P(NB \leq 0)] \\ \text{with } P(NB > 0) = \left\{ \frac{\left[\Phi\left(\frac{B_a \beta_1 + \mu_e}{\sigma_e}\right) \Phi\left(\frac{X\alpha}{\sigma_E}\right) - \Phi\left(\frac{X\alpha - K}{\sigma_{NB}}\right) + \Phi_{e,NB}\left(\frac{-B_a \beta_1 - \mu_e}{\sigma_e}, \frac{X\alpha - K}{\sigma_{NB}}\right) \right]}{\Phi\left(\frac{X\alpha}{\sigma_E}\right)} \right\} \end{aligned} \quad (16)$$

with μ_e , σ_{NB} , ρ and K as defined above, and with $P(NB \leq 0) = 1 - P(NB > 0)$. For computations using $NB2$ instead of NB , we simply replace B_a and RB by $B2_a$ and $RB2$.

B- The Data

1- A Look at the FES Data

To carry out our econometric analysis, we will use the original sample of 1985 FES data described in Chapter I; we will, however, restrict it in such a way as to eliminate from the sample all those units who are necessarily excluded from

claiming SB, that is, those where at least one of the adults works more than the maximum weekly hours specified (24 for a single parent, 30 for a couple) or for which savings exceed the £3,000 limit. This leaves 1515 income units, of which 503 report to have successfully claimed SB.

There are 59 observations of people who report having successfully claimed supplementary benefits but who are excluded from our sample on account of excessive savings or hours of work. This is probably mostly explained by the discrepancy between the conditions under which an income unit has claimed SB (e.g., working less than the specified amount of hours) and those under which it finds itself at the time of the survey. Although income units have the duty to report any change of circumstances to their social security office, they may fail to do so, they may believe this change to be temporary, or they may simply delay before complying with the regulation. Conversely, there probably are in our sample several income units who have just qualified for SB and who may not have had the time or opportunity yet to take it up. It is also likely that there exist inaccuracies in reporting the level of savings and hours of work analogous to those in reporting various income components -- which implies that the exclusion method just described will be subject to errors too⁵. The FES contains little information on savings as such, and the £3000 limit must be mostly applied using a very approximate grossing-up of investment income. To account for these inaccuracies and to avoid discarding from our analysis those 59 observations that

⁵ As discussed below, a failure to account for these errors will often lead the analyst to exaggerate the inconveniences to claiming a state benefit and thus to underestimate the rate of take-up. This is conceptually identical to the impact of the presence of errors in the measurement of B_g for which we are able to correct here.

declare a receipt of SB, we could add additional "hurdles" for eligibility to SB, hurdles that would be provided by the SB regulations on savings and hours of work. Since neither the government agency nor the analyst monitor and compute perfectly well units' data on labour supply or on savings, units that are dismissed here could thus find their way into the analysis. This additional feature would, however, involve extra computational complexity and it was not incorporated here, although it also clearly matters greatly to the overall assessment of the state support's allocative efficiency.

Table 4.1 shows the collection of data according to eligibility and claim; slightly more than 10% of the total of units which report claiming SB have negative calculated entitlement, that proportion rising to 16% for pensioners. Our sample of self-employed eligible to claim benefits is very small, and the results for that group might therefore lack precision.

On Table 4.2 we indicate the frequency distribution of claimants and non-claimants by amount of calculated entitlement. For both of these groups we show absolute frequencies in the sample, frequencies relative to the respective sizes of the subsamples of claimants and non-claimants, and the cumulative distribution (in %) of each subsample. We confirm that about 10% of the subsample of claimants are allocated negative calculated entitlement, and that the relative frequencies of the claimants with such entitlement discrepancies generally increases as we move up to zero calculated entitlement. A large number of non-claimants have a calculated entitlement that is both positive and close to zero (18.3% of them between £0 and £10 per week, for instance) and there arises therefore the inevitable suspicion that entitlement discrepancies similar to those

identified in the sample of claimants may have led to an erroneous assessment of SB eligibility as appraised by the DSS.

The last line of Table 4.2 computes take-up rates for each appropriate range of calculated entitlement. Take-up rates for those with negative calculated entitlement cannot, in this table, be assigned rates of take-up since we have not yet assessed econometrically the distribution of systematic and random entitlement discrepancies between our evaluation of eligibility and that made implicitly or explicitly by the DSS. For those units with positive calculated entitlement, we observe that as calculated entitlement increases, so does the apparent take-up rate, except for those upper ranges of entitlement where random fluctuations and low subsample sizes make the estimated take-up rates vary irregularly with entitlement ranges.⁶

2- On Grossing-up the Data

We might also be interested in knowing how our sample data compare to the official statistics produced by the DSS. For this, we consider Tables 4.3 and 4.4, where we have also provided separate results for pensioners and single parents, in line with the DSS's published figures. Looking first at Table 4.3 suggests the existence of a rather weak concordance between the two sets of population statistics that is reflected among other things in the difference between the estimated SB take-up rates. There are several reasons that may explain such

⁶ We shall see later in this chapter, however -- and we discuss more extensively in Duclos (1992c) -- that even those units with negative computed entitlement can straightforwardly be assigned a probability of claiming as well as one of being in receipt of SB given that they are deemed by the DSS to be entitled to it. In Duclos (1992c) we also exhibit the extent to which the unaccounted presence of random entitlement discrepancies in Table 4.2 exaggerates the expected positive relationship between entitlement and take-up rates.

divergences.

First, the DSS makes special adjustment to its FES data on the basis of the Department's own statistical enquiries; the most prominent of such adjustments is probably that on the subsample of pensioners, as stated above, and for which it is suggested that there are approximately 500,000 non-declared SB recipients. Accounting for such a number in our reckoning of pensioner recipients and eligible non-recipients roughly harmonises our figures with those of the DSS.

Second, the accurate estimation of weights for a subsample of relatively poor units requires information that is not readily available, and because such information may not be adequately transmitted by the use of weights obtained for the units of the whole sample, it is not surprising that our grossing-up procedure may not yield satisfactory results for the particular subsample of actual and potential SB claimants on which our attention is focused here. This would explain why the grossing-up of values for single parents does not add up to quite the same figures as those available from the DSS; we also recall that similar difficulties were encountered in the comparison of grossed-up receipts and recipients of one-parent benefits in Chapter I.

Third, the distinct population values obtained under the column "eligible but not receiving" may simply reflect structural differences in the information set used to generate the DSS and our own grossing-up weights. Our own sample statistics are gathered from the use of about 6 months of relevant FES data, and are grossed up using adjusted annual weights derived for the whole of the FES

sample⁷. To obtain "grossed-up" figures from FES sample statistics on the apparently eligible but non-recipient population, the DSS can use information from its own statistical records on the potential SB population; from this, it is (in principle) possible to compute some relatively more accurate grossing-up weights that reflect better the differential representation of population characteristics in the surveys, especially among the poorer. Whether this potential source of additional grossing-up information was appropriately tapped is, of course, a different matter.

Table 4.4 portrays information similar to that of Table 4.3, except that it deals with monetary statistics of the SB claims and failures to claim. We indicate the values of claimed benefits using either the amount of calculated benefits for those declaring receipts (shown in parentheses) or the declared amount of SB receipt (shown in brackets). As we will find later on in this and the following chapter, those two bases on which to compute SB receipts do not necessarily ought to yield similar values, and there are good factors (i.e., the presence of structural and random entitlement discrepancies) for which they should not. We note, incidentally, that the average amounts of claimed and unclaimed benefits vary widely between population groups, reflecting the variety of distributions of entitlements and burdens to claiming. There are again sizeable discrepancies between our global figures and those of the DSS -- reflecting the points made above -- but averages, which tend to throw some light on the characteristics of individual sample observations, are much closer. This suggests again that it is the grossing-up procedure that differs, and it raises issues of sample under-

⁷ On the derivation of these weights, see Atkinson, Gomulka and Sutherland (1988).

representation and differential response rates that are outside the scope of this study.

3- Differential Sampling and the Consistency of Estimates

We might therefore be concerned that effective differential sampling of population groups may distort our econometric results. So long as the probability of a population unit appearing in the sample is independent of the distribution of the unobservable cost E (which seems a reasonable assumption), our estimation results on the inconvenience to claiming SB will be unbiased and will yield information unequivocally applicable to units outside the sample. This result will hold even if the probability of appearing in the sample is correlated with the distribution of any of the elements of the vector of cost characteristics X , as long as that probability is unrelated to that of observing a particular E .

A similar argument applies with respect to the distribution of ϵ , although less convincingly since it is likely that, for instance, the distribution of random modelling errors ϵ (e.g., from deliberate misreports of income data) be linked to the probability of appearing in the FES sample. But what matters of course, for our present purposes, is the correct estimation of the distribution of random errors ϵ given that a unit is sampled and modelled, not the estimation of the hypothetical ϵ population distribution of sampled **and** unsampled units. In other words, we wish to understand the determinants of our entitlement models for the units that do appear in the survey; of course, if differential sampling were to be altered or eliminated, say, the shape of our average sample would then change and so would likely the determinants of our modelling of entitlement. This also reminds us that our empirical valuation of entitlement discrepancies – though not

that of costs to claiming, if our sample of units has not self-selected itself on that basis -- will be related to the structure of the survey (the FES) in other ways. Better FES data on savings and needs, for instance, would almost certainly reduce the presence of entitlement errors generated by the analyst, just as better and more regular eligibility assessments by the DSS would curtail entitlement discrepancies generated by the government's allocative agency.

Hence, summing up: although differential sampling of the population may create grossing up difficulties, it will not hamper the inference of an estimated distribution of costs upon the population if differential sampling is independent of the distribution of E . Furthermore, since an interest in the distribution of modelling errors independently of a unit appearing in the sample does not seem particularly relevant, we will not be concerned by differential discrepancies between our sample and the population for our modelling of entitlement.

From here onwards therefore, we shall focus exclusively on our sample statistics, with the caveat that we may not translate them directly into population figures. Were the task of extending our empirical results to yield estimates on the whole British population eventually to prove desirable, all that would then be needed would be the application of relevant grossing-up weights onto the sample results we shall derive.

4- Costs and Entitlement Indicators

The vector of characteristics X related to the perceived annoyance of claiming SB incorporates variables which have been chosen to reflect the physical, psychological, sociological and informational factors that are likely to influence a unit's perception of the act of going to the local DSS office and asking for an

award of supplementary benefits. Many of these variables have, however, more than one influence, and their net effect may not be what might a priori have been expected. The presence of children, for instance, may increase the physical and the time costs of going to the DSS office, but it may also decrease the psychological sense of "guilt" and sociological stigma attached to receiving government support. Having children may also enhance one's awareness or likelihood to be aware of such government support, and it may also generate a lower "equivalent value" of the benefit once it is shared across the members of the family unit. It is also most important to keep in mind how some characteristics may be correlated with the length of time over which families of prospective claimants expect to receive the benefit. Hence, for instance, pensioners and disabled people who might consider seeking a state benefit otherwise onerous may nevertheless foresee a longer expected period of eligibility and receipt, this latter feature plausibly decreasing their average weekly cost of claiming.

Our choice of cost variables has also been influenced by the results of previous studies on the take-up of state benefits. On Table 4.7 -- to which we will come back shortly -- we note by a "*" those characteristics which provided take-up parameters statistically different from zero. We see that few studies found that several of these characteristics yielded significant parameters. On Table 4.5, we disaggregate our FES sample of claimants and non-claimants into groups of various characteristics and compute the take-up rate -- using as a measure the number of claimants divided by the number of those thought to be eligible⁸ -- for

⁸ This aggregate take-up rate measure is different both from those used earlier in the chapter and from those applied in many earlier studies of the take-up of state benefits. We shall examine its desirability later in the chapter and find that, among

each group.

The variability in Table 4.5 of the group take-up rates with respect to changes in group characteristics suggests that the determinants of the take-up behaviour may be strongly linked to the socio-economic features of units. Single parents display the highest take-up rate (96%) of all groups, followed closely by the group of those couples with children (93%). Couples without children have, however, a calculated take-up rate of only 78%. One therefore ought to wonder whether the presence of children may not decrease significantly the expected weekly costs to claiming SB. The number of adults in the unit also appears to matter: units comprising no more than one adult member have a take-up rate of 69%, sizeably lower than for couples with or without children. The type of accommodation in which a unit lives also appears to be a take-up indicator, those renting claiming by 8% less than those in some special housing (council, housing association or new town dwelling). Units with a head not in work do not, on average, appear to take-up SB by a larger proportion than a typical sample unit.

The apparent impact of the remaining characteristics of Table 4.5 is, however, much more delicate to interpret. Those older units (made of at least one adult above retirement age) and those in apparent receipt of NI basic pensions (BP) comprise a number of observations for which a confusion of a SB for a BP payment is suspected. Hence, we cannot yet tell by how much the low take-up proportion for such units (respectively 54% and 52%) arises either from benefit confusion or from claiming costs that increase with age and pensioner status.

other features, it is robust to the presence of random entitlement discrepancies made by the analyst. This topic is also considered in Duclos (1992c).

Those few self-employed units which feature in our sample yield a very low take-up rate of 25%; for such units, there exist, however, indications that income is underreported in the FES and thus that we have overestimated their entitlement to SB. Thus, we find it unfeasible to determine before the full estimation of our model the degree to which such a low take-up rate arises from greater claiming costs to the self-employed or from a misleading overstatement of their SB eligibility.

There remain, moreover, other reasons to doubt the validity and the strength of the suggestive links derived from an analysis of Table 4.5. By segregating units according to unidimensional characteristics, such tables cannot account for the empirical correlation that exists, say, between the feature of being a single parent and that of living in special accommodation. If living in such accommodation were more likely to ensure high take-up rates, the finding of higher single parent take-up rates in Table 4.5 could rather reveal a positive correlation between single parenthood and special housing tenure. Similarly, being a single parent may be associated on average with greater SB entitlements, and the computation of high take-up rates for that group might simply stem from the greater financial appeal of SB not from any lesser claiming costs associated with single parenthood.

With these warning remarks in mind, one would like to test whether the features listed on Table 4.5 can, after a more elaborate statistical analysis, reveal take-up links that are both significant and congruous with the above preliminary indications. To describe the socio-economic characteristics of the income units, we thus use the number of dependent children (CH) and how many of these are

under the age of 5 (CH05), as well as the number and mean age of the adults found in the unit (AD and M(AGE,AGES), where AGES is the age of the spouse, where there is one). If SING=1 the unit is made of a single-parent family. If the head of the unit is also the head of the household (which may contain more than one eligible income unit or "family"), then HEAD=1; such a distinction may clearly matter if being a head of household also entails privileges or responsibilities (e.g., looking after the dwelling) which might be somewhat avoided by other units. We also include the type of accommodation in which the unit lives: COUNCIL=1, if it is found in a council, housing association or new town corporation dwelling, and RENT=1 if COUNCIL=0 though the members of the unit are tenants.

The average of the age at which the adults ended their full-time education is indicated by M(ED,EDS), and the average of the hourly wages, by M(WAGE,WAGES). When appropriate (viz, for non-participants), the value of such wages was predicted and filled using the results of an estimation procedure over the whole FES sample, allowing and correcting for various interactions between the decision to participate in the labour force and the wage rate faced by members of the units⁹. Although the hourly wages so obtained are for many reasons likely to be rather inaccurate indicators of the opportunity cost of leisure, we nevertheless include them with the hopeful aim of detecting the value of a claimant's expected loss of time or leisure in claiming SB. Wages will, however, also be correlated with some of the units' observable and non-observable characteristics, and the estimated effect of wages on the costs to claiming may rather reflect the impact of these characteristics. We also incorporate variables that

⁹ The procedure and the results are described in Duclos (1992b).

denote the absence from the labour force of the head of the unit: UNO=1 and BPR=1, if the head is not working or is a declared National Insurance pensioner, respectively. WKAB shows the number of weeks which the head has been spending away from work, up to a maximum of 52. When SN=1, one of the adult members of the unit is self-employed. As a further variable we also include the log of YGROS, which is income gross of (excluding) SB; this might enable us to see if and how the monetary cost corresponding to the loss in utility from claiming varies with the level of original income, a query suggested by Cowell (1986) and related to the nonlinearity of the agents' utility function.

We must also specify the vector of variables which will form the basis of our analysis of μ_e . To allow for general under- or over-reporting of income and thus for over- and under-estimation of entitlement B_a relative to B_g , a unit vector (UNIT=1) is fitted. We also include SN and SN·YGROS to provide for the modelling of the entitlement of units with self-employed heads, as was described above.

C- Explaining the Empirical Results

1-Introduction and General Results

We can now present the results of the maximisation of the full likelihood function developed in chapter III and adapted above to the requirements of our FES sample. We can divide our estimates into 5 groups, corresponding to five prominent features of our analysis:

- Costs to claiming
- Entitlement indicators, μ_e

- Probability of a sub-group of pensioners faithfully reporting receipt of SB
- The effect of housing benefits upon the claims of SB
- Standard deviations of unobservable random effects

The estimated value of the relevant parameters, along with the respective asymptotic standard errors in parentheses, are shown on the two pages of Table 4.6. The first column of estimates shows the results obtained when no constraint is put on any of the parameters, except for the normalising of $\sigma_{NB}=1$. As discussed in chapter III, this nevertheless allows for the estimation of the standard deviation of costs relative to that of entitlement: a look at the last two lines of the second page of Table 4.6 shows that the ratio of the estimated standard deviations is $0.465/0.885=0.525$. This suggests that unobservable burdens to claiming are significantly less dispersed in our sample than unobservable and random divergences in modelling entitlement.

Five cost parameters appear statistically different from 0 at the customary 5% significance level: the ones for the number of adults (AD), for a head of household (HEAD), for the square root of weeks spent outside work (SQRT(WKAB)), for the mean hourly wage (M(WAGE,WAGES)), and for the log of gross income (L(YGROS)). Apart from that on HEAD, all such parameters are, on closer scrutiny, quite comfortably away from zero. That no more of the estimates appear, in fact, significantly different from zero must stem partly from the large number of the cost parameters (15 in total, including the cost constant term) and from the multicorrelation that prevails among several of the cost and entitlement characteristics. The "AGE" variable, for instance, is highly correlated (i.e., correlation coefficient of above 0.5) with HEAD, BPR, BP and UNO, whereas

SN is similarly correlated with entitlement B and B2 , UNO and YGROS. We have nevertheless opted to leave the set of the original cost variables largely untouched, even if removing a few of them might have generated statistical significance for some of the remaining others. The parameters pertaining to the entitlement vector RB are not significantly different from 0. Let's note in particular the highly significant parameter on calculated entitlement B_a and $B2_a$, which confirms the report of all studies -- except that of Altmann (1981) for male pensioners -- that the take up of benefit is strongly related to the level of entitlement.

BPROB, describing the probability of **some** pensioners confusing a SB payment for a receipt of a retirement pension, has a parameter very significantly different from zero; its numerical value, 0.465, suggests that there is almost a 50% probability for such pensioners that a confusion of benefits will occur if SB is received. One reason for which this parameter of BPROB might have been overestimated stems from our implicit assumption that, for a pensioner not in declared receipt of SB and for whom it was not possible to be reasonably certain that a benefit confusion had not occurred, a declared receipt of retirement pension is either **fully** legitimate or **fully** misreported. A pensioner unit truly receiving the two benefits might be better able (more likely) to distinguish the two benefits than a unit receiving only one of the two, and therefore we might expect that a pensioner unit already in receipt of a pension payment would not confuse a separate payment of SB with it. The danger still exists, however, that for those for whom $BPROB=1$ only part of the reported pension is in fact a SB payment, in which case our calculation of $B2_a$ would be mistakenly large, causing in the econometric process the estimate on BPROB to be biased upwards.

The increase in the award of housing benefit whenever supplementary benefit is claimed seems to have no discernible empirical effect on the take-up of SB: the parameter β_2 on BHCS is at its lower bound, 0. Thus, either BHCS suffers on its own from severe errors of computation, in which case its inclusion in the econometric process cannot yield the desired empirical information, or income units simply do not significantly take into account the (admittedly relatively small) related change in housing benefit when they consider claiming SB.

2- Sources of Randomness and Divergences in the Computation of Entitlements

We can now impose on our model the previously discussed constraint that the parameter on the calculated entitlement variable (B_a and $B2_a$) is one. To impose the normalisation, we simply divide all the estimates (including the standard errors) on the first column of Table 4.6 by 0.0462, the estimated parameter on B_a and $B2_a$, and we eliminate the normalising constraint that $\sigma_{NB}=1$.

This procedure yields **absolute** estimates of the standard deviations of our error terms, E and ϵ : this is shown on the last two lines of the second page of Table 4.6. Unobservable costs to claiming have an estimated standard error of £9.66 per week, while that of unobservable modelling errors is slightly more than £19 per week. Hence, the unobservable variation of net benefits across our sample appears to be mostly due to the presence of random errors of modelling; the untruncated normal distribution of NB has an estimated variance $\sigma_{NB}^2 = \sigma_E^2 + \sigma_\epsilon^2 \equiv 464$, with a corresponding standard error of about 21.5. We must guard ourselves, however, against forgetting that σ_ϵ does not necessarily reveal information on the extent of random errors stemming from the use of the FES

income. As we have already said, ϵ should indeed be seen in the context of our simplifying assumptions as the difference between the error made in our own FES calculation and that made at the DSS; that is,

$$\epsilon = \epsilon_g - \epsilon_a \quad (17)$$

The standard deviation of ϵ is then:

$$\sigma_\epsilon = \sqrt{\sigma_{\epsilon_a}^2 + \sigma_{\epsilon_g}^2 - 2 \cdot \text{cov}(\epsilon_a, \epsilon_g)} \quad (18)$$

This reveals that σ_ϵ can in fact either underestimate or overstate the degree of random miscalculation using (and random misreport in) the FES data. So, for instance, if, on the one hand, errors made at the government agency (DSS), ϵ_g , are uncorrelated with those we make using the FES, ϵ_a , then $\sigma_{\epsilon_a} < \sigma_\epsilon$ and σ_ϵ overestimates the spread of the analyst's entitlement errors; if, on the other hand, ϵ_g and ϵ_a are sufficiently positively correlated, it is possible that $\sigma_{\epsilon_a} > \sigma_\epsilon$, and the estimated variance of ϵ will underestimate that of ϵ_a .

Our procedure also generates values of entitlement parameters which can be readily interpreted, although none of them are statistically significant from zero by the conventional 5% criterion. The intercept (UNIT) has an estimated parameter which suggests that, relative to B_g , we underestimate by about £4 the entitlement of our units as assessed by the government agency. This relative underestimation could partially be attributed to our inability to account for various supplementary requirements or needs which enter SB but which cannot be imputed from the FES. It could also stem from differences between the report

and accounting of income data in the FES and in the DSS offices. Prospective claimants might conceal more income when they meet their social security officer than they do when prompted by the FES interviewer, or their DSS officer might be on average more lenient and less inquisitive than the SB rules seem to demand. This relative DSS generosity is intuitively also consistent with the expectation that units will on average reveal more readily to the agency changes in their situation that augment their SB entitlement than changes that do the opposite.

For self-employed units, our estimates suggest that we may generally apply arguments opposite to those just stated. Using the model of entitlement for self-employed developed above, we note that the (net) fixed effect of dealing with a self-employed unit when attempting to establish its entitlement B_g is that of overestimating it by an amount of £23.9 per week. Furthermore, from the value of the parameter on SN*YGROS, we find that the point estimate of b in

$$YGROS_g = b \cdot YGROS_a + a - \eta_a \quad (19)$$

equals $b=1+0.437=1.437$, which implies that the absolute level of overestimation ($YGROS_g - YGROS_a$) increases with reported gross income. We must, however, wield the latter estimates with care since they are clouded with imprecision. This comment seemingly applies to all our results regarding the self-employed, for whom the sample of relevant observations is very small.

3- On the Costs to Taking up

We now turn to the determinants of what we have called the "costs to claiming" SB. As mentioned earlier, we are not able to segregate empirically the impact of characteristics upon different types of costs, e.g., the effect of having

children on the separate physical and psychological inconvenience of taking up. This being said, we also note that the burden implied by many characteristics is not different from zero at reasonable statistical significance levels, and that we might therefore take it with a grain or two of salt. We must also keep in mind that the parameter estimates of Table 4.6 cannot always be read accurately as estimates of the expected burden of claiming. This is because the normal distribution of E is truncated below $-X\alpha$, and that expected take-up costs equal:

$$\begin{aligned} E[X\alpha + E] &= X\alpha + \frac{\phi\left(\frac{X\alpha}{\sigma_E}\right)}{\Phi\left(\frac{X\alpha}{\sigma_E}\right)} \cdot \sigma_E \\ &= X\alpha + \lambda^*(c) \cdot \sigma_E \end{aligned} \quad (20)$$

with $c=X\alpha/\sigma_E$ and $\lambda^*(\cdot)$ being the complement of Mills' ratio. Thus the expected marginal cost of a characteristic X_i in the vector X is:

$$\frac{\delta E[X\alpha + E]}{\delta X_i} = \alpha_i \cdot (1 - [c \cdot \lambda^*(c) + \{\lambda^*(c)\}^2]) \quad (21)$$

where the term in squared brackets is always smaller than 1 and where the derivative thus has always the same sign as α_i . The expected marginal cost is largest (lowest) when c is large (low), and approaches α_i when c tends to infinity.¹⁰

¹⁰ Similarly, the variance of the truncated distribution of E can only asymptotically be that of the untruncated normal, σ_E^2 , and will generally be lower than the latter. Using integration by parts and the fact that $\delta(\phi(x))/\delta x = -x\phi(x)$ we can find that the variance of the truncated distribution of costs equals $\sigma_E^2(1 - [c\lambda^*(c) + (\lambda^*(c))^2])$.

Hence, we may keep in mind that the estimates shown on the last column of Table 4.6 overestimate at least slightly the expected marginal burden to which the corresponding elements of *X* are associated. It is also instructive to compare our qualitative results with those of earlier studies -- which provided evidence on the effect of various socio-economic characteristics on the probability of claiming state benefits -- and this is done in Table 4.7. A "+" ("-") appears when a characteristic is associated with a greater (lesser) probability of taking up. In contrasting our findings, we ought to bear much more in mind those results which were found to be statistically significant. To this effect, we indicate by a star those parameters estimated to be different from zero at a conventional 5% level. "NA" shows that the effect of the characteristic could not be adequately inferred from the results of the studies. Furthermore, only two inquiries are really directly comparable with our own examination: the analysis of Altmann (1981) on the claims of SB by male pensioners, and that of Fry and Stark (1987) on SB claims for separate groups of pensioners and non-pensioners. We therefore separate them in Table 4.7 from the other studies and will wish to pay special attention to their peculiar comparative findings.

Our results show that, to a unit already saddled with sizeable claiming costs, the addition of a dependent child would entail costs lower by about £4 a week on average, this effect being insignificantly different from zero. Psychological reasons for the existence of such a lower burden can easily be found, such as a lesser sense of guilt and a more intense feeling on the part of the income unit that the benefit is then a "due". Such factors would then outweigh the greater physical burden of requesting a state benefit when one must care for

children. This lower inconvenience to claiming in the presence of children also appears to be supported by the evidence of previous studies, although none reported statistical significance.

The presence of an extra adult can augment the claiming inconvenience by up to £32.20. Furthermore, this strong effect is also significantly different from zero and adds to the result of Dorsett and Heady (1991) on housing benefits, although it finds no support among the previous SB inquiries. Thus, families with several adults will, other things being the same, be much less likely to take up Supplementary Benefits. This may be due to the presence in these units of a stronger sense of stigma and uneasiness when state support is sought. Alternatively, since eligibility to SB depends on the combined income of all adults and on their **joint** absence from full-time work, it may be that units with several adults expect the costs of taking-up to be spread only over a relatively short period (e.g., until one of the members finds full-time work), thus leading to relatively high average weekly costs.

Being a single parent will entail sizeably lesser costs (a fall of up to £10.8) than those faced by two-parent families, especially so if we were also to incorporate the impact of AD (the number of adults) just discussed. This statistically insignificant result would tend to confirm the earlier result of Fry and Stark on the claims of SB by non-pensioners. It provides another example of how a characteristic can be associated with widely different impacts on the perceived burden of claiming a benefit. The presence of more than one adult should generally make it physically easier for the unit to apply for SB, and should therefore limit the trouble of taking up; a unit with only one adult may yet feel

that it deserves state support more than other units, the psychological stigma and uneasiness of going to the DSS office might be so much the lesser, and the expected time of eligibility and receipt may be much longer.

Becoming older or being a head of household both shrink the perceived costs and reluctance of taking up SB. These two impacts hover around the 5% level of conventional statistical significance. Becoming older may signal a lesser dislike of state support, and can be correlated with a greater expected period of benefit eligibility. Alternatively, it can hint to a greater awareness of the availability of state support and of a poor person's "right" to it, though this (statistically insignificant) result is not supported by most previous studies. The parameter on HEAD is particularly large, which, as suggested above, might imply that a head of household considers his set of responsibilities and needs to be sizeably enlarged, decreasing the perceived inconvenience of requesting the state's support. Both of these results support the statistically significant findings of Altmann (1981).

Being more educated bears no statistical significance and very little absolute impact on the cost to taking up. The same can be said of the effect of living in a council, housing association or new town corporation dwelling (COUNCIL=1). Renting a private accommodation, though not yielding statistical significance, is associated with a sizeable additional burden to claiming, in the order of up to £10.9 per week. That these variables do not generate significant results seems unsurprisingly to reflect the mixed evidence of the previous studies and fails to replicate the significance of housing in Altmann's enquiry, based as it is on a pension subsample.

Work status appears to provide important take-up determinants. Being outside paid work entails statistically insignificant weekly costs of up to £6.30, but a very significant effect is that of the number of weeks spent outside work: a stretch of 16-week unemployment may cut average weekly costs to claiming by up to £35, either by spreading the take-up costs over a longer period, or by making the information and opportunity costs of seeking state support lower, or by being correlated with other unit characteristics which denote a lower claiming burden. Statistically insignificant are the self-employment and receipt of a National Insurance pension parameters, indicating extra costs respectively of up to £12.80 and £6.32 per week. Furthermore, it is not practicable to compare our estimate of the impact of pensioner status on SB take-up since all relevant former studies have applied separate analyses on the subsamples of pensioners and non-pensioners.

The monetary variables included in the analysis of costs (hourly wages and log of pre-SB income) yield not wholly expected results. The coefficient on the mean of wages is negative and significantly different from zero, implying either that the variable is a poor indicator of the opportunity cost in leisure to those more likely to contemplate taking up the benefit – the majority of whom are out of work – or that the parameter picks up a correlation between, say, the ability and awareness of units and the value of their (mostly potential and predicted) wages. A similar comment applies to the estimated negative and significant impact of $L(YGROS)$ on the trouble of claiming, an impact which is the opposite of what a model with decreasing marginal utility of income would a priori suggest. Greater original income may, however, be linked to greater physical and

informational ease to taking up SB. This unit income result nevertheless contradicts the apparent findings of Fry and Stark on SB take-up and of Blundell, Fry and Walker on HB take-up. It is worth recalling, however, that such studies did not account for random entitlement discrepancies in understanding the take-up of state benefits. Because of this, they had to justify the presence of apparently low take-up rates at low computed entitlements partly through the generation of statistical biases on cost parameter estimates. It is thus not entirely surprising that they attribute a negative parameter estimate to a variable (household or unit income) that is negatively correlated with computed entitlements.

Table 4.8 gives an overall impression of the estimated magnitude of expected costs involved in claiming a benefit and of the considerable variation that seems to prevail between units of different characteristics. For the three reference units depicted, weekly expected costs range from a little less than £1.50 for a 6-month unemployed single parent to up to £30.30 for a family of four of whom one member is in self-employment. As we have discussed, expected costs are greater for the younger, the tenants and the self-employed, and lower for those with children, the single parents and the one-adult units.

From our estimation of the parameters of the likelihood function, we can readily illustrate probabilities that units will claim SB, given their calculated entitlement and their observed characteristics. For this, we need to compute the expected take-up costs involved (using, say, Table 4.8 as a guide), and we also require the calculation of the unit's entitlement B_g as assessed, on average, by the government agency (equal, for $BPROB=0$, to $B_a+RB\bullet\beta_3$). We then simply plot the two values obtained on Figure 4.2, which show the estimated probability

(PROBTAKE) that a unit with expected agency entitlement "ENTLMENT" and expected costs "COST" will claim SB. Thus, if we establish that the entitlement B_g of a unit is, say, expected to equal £28 per week, and that its observed characteristics lead to expected costs to claiming of £30.24, Figure 4.2 readily tells us that there is yet a 45% chance of the unit claiming SB, allowing for unobservable costs and random errors of modelling entitlement. We also note that even those with negative expected entitlement B_g can display a sizeable probability of claiming SB, especially if they exhibit characteristics that hint to a low inconvenience of taking up.

4- On Type I and II Errors

We have already stressed that our simplifying assumptions made for the purposes of estimation are not sufficient to reveal unbiased estimates of B^* . In general, we cannot say anything on the value of B^* ($=B_a - \epsilon_a$) since we only observe B_a and our estimates of the distribution of ϵ combine the unknown estimates of the distribution of ϵ_a and that of ϵ_g . For illustrative purposes, however, we will assume in this short section that $\epsilon_a \equiv 0$ and therefore that $B_a \equiv B^*$: the analyst always computes accurately the true level of entitlement to a benefit. Such an assumption is of course unrealistic since it implies that all entitlement divergences ϵ stem from administrative errors made by the government agency, but it is helpful since it allows the computation of the distribution of B_g when we can measure B^* . We may then illustrate the impact of administrative errors on the level of the Type I and II errors introduced Chapter III.

A Type I error occurs when a unit that is truly entitled is not considered eligible by the administrative agency. Conversely, a Type II error arises when a

unit requesting a state benefit is wrongly considered eligible by the agency. In Table 4.9, we consider the "potential" level of each error type as well as their actual "occurrence" under the assumption that $B_a \equiv B^*$. "Potential" levels of type I and II errors refer to those that would be achieved by the agency if all units were assessed or, equivalently, if units faced no costs to requesting the state benefit. Hence, the first column under "Type I errors" reveals that, if all units were to apply, there would be on average a conditional probability of 18.1% that a unit be mistakenly refused eligibility when it is truly eligible ($B^* > 0$), leading to wrongly retained benefits equal to 6.68% of the total benefit payable by the agency and to a group of the size of 23.2% of recipients not duly receiving state support. If all units were to apply, there would exist a 18.8% average conditional probability that a unit not truly entitled be mistakenly granted some state support leading to a correspondingly greater relative cost (7.55% of total benefit payable) in benefit levels. On this *benefit* basis Type II errors would thus appear potentially slightly more costly than type I errors. When we consider the *number* of mistaken cases, however, type I errors exceed somewhat those of type II.

In practice, units also face claiming costs which limit their utility to requesting a state benefit. Thus, given that a unit is truly eligible ($B^* > 0$), there exists a conditional probability of 30.2% that it does not receive the state support. The difference between the "occurrence" and the "potential" ($0.302 - 0.181$) shows the impact of inconveniences to claiming upon the desire of a truly eligible unit to take up the benefit. The cost of such type I errors -- both in the number of mistaken cases and in the sum of the benefits not granted to truly eligible units -- is also increased when we account for inconveniences to claiming. Conversely,

costs to claiming reduce the likelihood of a Type II error arising because fewer "impostor" units then find requesting the benefit worthwhile; in other words, inconveniences to claiming then act as a screening device. Comparing the values for Type II errors under the column "Potential" to those lying below "Occurrence", we find that costs to taking up reduce the conditional probability of a Type II error arising from 0.188 to 0.098, and the level of mistakenly granted benefits decreases from 7.55% to 5.45% of the level of total benefit payable by the agency. The number of mistaken Type II cases also falls sizeably.

As discussed by Cornia and Stewart (1992) on the basis of data across a number of developing countries, the pursuit of a low level of mistakes of one type would thus characteristically spur the appearance of errors of the other type. The imposition of sizeable deadweight losses will therefore be optimal if the state's objective function does not discount much the size of gross benefits by the level of claiming costs incurred, but finds it explicitly or implicitly undesirable to grant help to those not truly deserving it. In this case, the government may well find it best to keep claiming costs high such as to keep the probability of Type II errors low. The optimal setting of claiming costs is then the outcome of a trade-off between decreasing the "occurrence" of Type I errors via a lower inconvenience to seeking the state's support, and decreasing the "occurrence" of Type II errors via a rise in the level of the same inconvenience. We will come back to this issue in the next chapter.

5- On the Fit of the Model

Before we go on to throw some light on various take-up statistics, we might want an indication of how well our model fits the data of non-pensioners and

pensioners alike. This can be seen from Figure 4.3a, which shows the cumulated number of observed and predicted claims from non-pensioners along the axis of the centiles of calculated entitlement B_a . The solid curve A shows the SB claims as observed in the FES, whereas the four others indicate claims predicted by the model under various assumptions. Hence, the second (B) curve displays the proportion of claims that would be obtained if we were to predict take-up on the basis of calculated entitlement only, with no costs to claiming and no adjustments for confusion of benefits. Under such assumptions, anyone calculated from FES data to have positive entitlement to SB would be predicted to register a SB claim. Curve B thus shows as a straight line as soon as calculated entitlement becomes positive.

To curve B we can then add curve C which exhibits the cumulated number of predicted claims of a unidimensional Probit model in which observations of claims and non-claims are explained solely by "modelling" errors and divergences in the computation of entitlement, using the entitlement indicators (B_a , $UNIT$, SN , $SN*YGROS$) of the complete model. That is, we run a simple Probit model using as exogenous variables those that appear in the vector of our entitlement indicators. Although such a model assigns observations of claims and non-claims to pure random errors in our computed level of entitlement and thus assumes no cost to claiming, it is *structurally identical* to all the models of Table 4.7 and serves to show once more the undesirable implicit design of previous enquiries into the allocative efficiency of state support. This restricted model easily fails a likelihood ratio test against the full model at a 1% statistical significance level¹¹, but curve

¹¹ The log-likelihood of this restricted model equals -567.98.

C nevertheless explains remarkably well the cumulative distribution of claims, especially in comparison to the full model depicted by E. Thus, if a relatively efficient curve fitting (as opposed, say, to good explanatory power) of the take-up of SB were desired, a univariate Probit could be run on all observations and **all** apparent failures to claim could be straightforwardly attributed to random and systematic errors in calculating entitlement. An immediate corollary of such a simple analysis would be the finding of 100% take-up rates if the basis of eligibility were that made by the DSS. The removal of all entitlement discrepancies would, under such assumptions, also lead to the observation of perfect rates of take-up, a feature which is, again, not intuitively very sound. Because of its structural lacunae, the model would moreover not be fit for assessing either the aggregate or the individual impact of tax and benefit reform.

We can also set a curve D displaying the prediction of claims using the estimates of the full model but subsequently assuming for predictions that there are no costs to claiming SB. The gap between curves D and E then illustrates well the impact of the estimated inconvenience to claiming upon the take up of state benefits. This graphical fit of E, the predicted claims using our full-fledged model, to A, the registered distribution of claims, is not perfect. The two curves follow each other fairly well throughout the range of entitlement, but there are some divergences around the level of 0 calculated entitlement when the model overpredicts the cumulative take-up of SB.

Figure 4.3b replicates the above procedure for the subsample of pensioners. There curve B, built from attributing predicted claims to the pure computation of entitlement, does not follow well the distribution A of registered claims. As

expected, estimated costs carve a widening wedge between curves D and E. A statistical model built solely on ascribing claims to calculated eligibility and random and systematic entitlement divergences would yield curve C of predicted claims; this would do no worse in the task of *curve fitting* than our full model E, which allows for benefit confusion and which consistently overestimates the cumulative declaration of claims A.

D- On the Statistics of Benefit Take-up

We will, in this section, focus on some of the issues surrounding the rate of benefit take-up, i.e., the proportional importance of benefit receipts relative to benefit entitlement. We shall first consider how the rate can and should be defined empirically under varying assumptions and definitions¹², and we will then examine the ways in which our econometric findings might provide evidence of the take-up of SB.

The rate of benefit take-up features prominently in analyses of the efficiency and equity of social security and state support¹³. It is clear, however, that a take-up rate has little analytical value; as a single indicator, it cannot but describe poorly the joint distribution of costs to claiming and benefit entitlement which is the essential structural tool to explain the impact of a government's intended support. As Fry and Stark (1987) point out, it is also obvious that the rate of take-up is not invariant to changes in the distribution of entitlement – or,

¹² Issues surrounding the definition of take-up in the context of state benefits (such as FIS and HB) that are structurally different from SB are described in Duclos (1992c).

¹³ See, for instance, on that topic the official 1985 government response [e.g., DSS (1989)] to opposition critics regarding the Supplementary Benefit, Family Income Supplement and Housing Benefit schemes, made public by the DSS.

for that matter, to changes in the distribution of the burden of claiming -- and thus to changes to government policy. We will consider this last point in greater detail in the next section.

The principal aim, in defining the index, must be to assess as accurately as possible the set of those claiming **and** that of those eligible to claim. One must also specify whether the interest is in those who are in principle positively entitled ($B^* > 0$) or in those who would be deemed by the government agency to be eligible ($B_g > 0$). For the purposes of the exposition we shall initially assume that $\varepsilon_g \equiv 0$ and thus that $B^* \equiv B_g$. All errors of modelling ε are thus made by the analyst and it is then possible to compute unbiased estimates of B^* . We shall subsequently see how the optimal definition of the take-up rate might change when this assumption does not hold and when our estimates of ε also subsume administrative errors made by the government agency. We finally provide some summary results based on our econometric estimates.

1- Absence of administrative errors ($\varepsilon_g \equiv 0$) or take-up statistics based on administrative eligibility $B_g > 0$

We first assume that B^* is exactly assigned by the government agency when assessing the entitlement of units. The results of this section should hold analogously if we simply wanted our take-up statistics to be based on administrative eligibility ($B_g > 0$). By displaying the magnitude A to E of various sets, Table 4.10 shall help us consider various empirical definitions of the take-up rate, definitions which invariably involve the ratio of the size of a set of claimants over the size of a set of the eligible.

To evaluate the magnitude of these two separate sets, the habit of the DSS

has been to combine its own statistical enquiry to estimate the number of claimants, A , and to add to it the number, C , of eligible non-claimants as estimated from the FES, yielding $A/(A+C)$ as an estimate of the rate of take-up. We stress that, in contrast to us, the DSS is thus able to assess the number of claimants A from an alternative data set, whereas we must rely on the FES to provide information on both the set of claimants and the set of the eligible. The soundness of the DSS's procedure clearly depends on the accuracy of the Department's enquiry but also on the extent to which "errors of modelling" ϵ_a are made in the calculation of eligibility. Assuming that the two sources of information are broadly equivalent (i.e., in the limit the information which they would commonly provide on the set of claimants is identical), the DSS estimates will likely underestimate the actual rate of take-up, since the size $(A+C)$ will include all those calculated to be eligible plus some units which would have been calculated to be ineligible recipients but which still appear in the denominator as entitled recipients since they declare claiming SB. In other words and for $\mu_\epsilon \leq 0$, $(A+C)$ overstates the number of the eligible since it includes through A all those considered eligible by the government agency (but whom we may not have thought to be eligible) but fails to exclude from B those whom the agency would not have considered eligible (because we believe them to be eligible). Thus, even if A might provide an accurate estimate of the number claiming, adding A to C would overestimate the number of the eligible in the presence of purely random modelling errors ϵ_a . This is what would happen in Table 4.10, where we have assessed A using the FES; there, $(A+C)$ overestimate in normal circumstances (i.e., in the absence of systematic and adverse entitlement errors μ_ϵ) the size of the set

of eligible recipients.

Fry and Stark (1987) propose a different empirical definition of the take-up rate, which relies on the exclusive use of FES data and which they consider "to be a natural definition -- namely the proportion of those thought to be entitled to benefit who actually claim it" (p.5). They thus suggest using $B/(B+C)$, the ratio of those claiming **and** thought to be eligible over those thought to be eligible. However, in the circumstances described above¹⁴, the measure $B/(B+C)$ proposed by Fry and Stark does not help matters, for it further underestimates the take-up rate. Whereas $(B+C)$ may estimate better than $(A+C)$ the size of the set of eligible units, using B instead of A will generally underestimate the set of those claiming since it makes the status of claimant additionally conditional upon the status of calculated eligibility to SB. The correct estimate of an individual's "take-up" rate, based on B^* , is given by $P[NB \geq 0 / B^* \geq 0]$. When $B_g \equiv B^*$, this conditional probability expands into

$$P[NB > 0 \mid B^* > 0] = \frac{P[NB > 0 \wedge B^* > 0]}{P[B^* > 0]} = \frac{P[NB > 0]}{P[B^* > 0]} \quad (22)$$

To obtain an estimate of the "aggregate" take-up rate, one would need to divide the sum of all numerators by the total of all denominators. One would then obtain an asymptotically correct estimate of the size of the set of claimants over that of the set of the eligible. By making the relevance of claims dependent upon computed eligibility, Fry and Stark asymptotically sum the following numerators:

¹⁴ That is, $\epsilon_g \equiv 0$ (no administrative errors) or when take-up is measured on the basis of DSS eligibility B_g .

$$P[NB>0 \wedge B_a>0] \quad (23)$$

which are always strictly less than $P[NB \geq 0]$ whenever the probability of making random modelling errors ϵ_a is not null. The more variable is B_a around B^* , the more will $B/(B+C)$ understate the take-up rate when $\epsilon_g \equiv 0$. Since B is always no greater than A , the measure $B/(B+C)$ will underestimate the take-up rate further than the measure $A/(A+C)$ proposed by the DSS. This can also be checked in Table 4.10, where the aggregate take-up rate falls from 66.5% when defined by $A/(A+C)$ to 64.2% when described by $B/(B+C)$.

The best estimate of the aggregate take-up rate that one could possibly provide, in the absence of an econometric model of entitlement and costs to claiming, is $A/(B+C)$ -- possibly making, as much as is reasonably possible, ad hoc corrections for deficiencies in sampling and reporting procedures. This is because the best we can then hope is that A will be asymptotically equal to the population size of the set of claimants and that $(B+C)$ would reasonably yield the best estimate of the number of the eligible. *Ceteris paribus*, there are then no *a priori* reasons for preferring B to A or $(A+C)$ to $(B+C)$. Using $A/(A+C)$ or $B/(B+C)$ would yield more accurate take-up estimates only by coincidence and only when systematic errors in ϵ_a are difficult to assess. In spite of the *apparent* awkwardness of including in the number A some units which are calculated not to eligible to receive the benefit, $A/(A+B)$ equals asymptotically the true take-up rate in the absence of systematic entitlement errors μ_e . As can be seen from Table 4.10, using $A/(B+C)$ rather than $A/(A+C)$ or $B/(B+C)$ can make an important difference to the estimated aggregate take-up estimates. As mentioned above, this is necessarily

true under the assumption of no administrative errors, $\epsilon_g \equiv 0$, or when we wish to measure the rate of take-up on the basis of eligibility as appraised by the DSS.

We can, however, do better in the circumstances and provide estimates of the sizes of the expected sets of those claiming and of those eligible. These are respectively shown in columns D and E, leading to the take-up rate D/E . D/E differs sizeably from $A/(A+C)$ and $B/(B+C)$, especially for subgroups such as pensioners and the self-employed. For pensioners, D/E attempts among other things to redress the "benefit confusion" bias which other studies would have likely tried to correct indirectly. For the self-employed, the increased take-up rate is mostly a cause of the revision of entitlement, allowing among other things for relative underreporting of income data. We also note that the "econometric" take-up estimate D/E is in the aggregate closer to $A/(B+C)$ than it is to the DSS's $A/(A+C)$ and to Fry and Stark's $B/(B+C)$.

2- Existence of administrative errors ($\epsilon_g \neq 0$) and take-up statistics to be based on true eligibility ($B^* > 0$)

The usefulness of our econometric estimates to evaluate the aggregate rate of take-up depends on how closely we can assess ϵ_a (entitlement errors made by the analyst). It also hinges on whether we are interested in the probability that a unit claims given that it is truly eligible or given that it would be considered eligible by the government agency. In the latter case, we can use our estimates of the entitlement as assessed by the agency, $B_g = B_a + \epsilon$, to provide estimates of the take-up rate even if we cannot appraise ϵ_a separately; an individual probability of take-up is then $P(NB > 0 \mid B_g = B_a + \epsilon > 0)$, where $\epsilon = \epsilon_g - \epsilon_a$ and where ϵ_a is not separately identifiable. In the event in which $P(NB > 0 \mid B^* = B_a - \epsilon_a > 0)$ is the desired measure,

the soundness of using $P(NB>0 \mid B_g=B_a+\epsilon_g-\epsilon_a>0)$ will rest on whether ϵ_g can reasonably be neglected. If ϵ_g as a relative source of entitlement errors cannot be sensibly dismissed, B^* differs significantly from B_g and none of the measures described above will necessarily yield better take-up estimates.

In the extreme case in which the analyst makes no errors in assigning entitlement, viz, $\epsilon_g \equiv 0$, Fry and Stark's $B/(B+C)$ would provide the best estimate of the aggregate probability of receiving SB given that a unit is truly eligible ($B^*=B_a>0$) and our econometric estimates D/E would yield the correct probability of take-up given that a unit is judged eligible by the government agency ($B_g=B_a+\epsilon>0$). The use of B in $B/(B+C)$ then succeeds in separating those who have been mistakenly granted SB and includes in the set $(B+C)$ of the eligible some units who would mistakenly not have been regarded as eligible by the agency. However, if agents base their decision to request the benefit not on the level of B^* but rather on a fairly accurate perception of B_g , using D/E even when $\epsilon_a \equiv 0$ then generates a better index of the costs and inconveniences to seeking SB.

3- Summary Results

It should by now be clear that our econometric model allows for the establishment of entitlement figures, B_g , which can be significantly at odds with those provided by our initial estimates, B_a . Table 4.11 displays the effect of the two most important factors, moving from B_a to expected eligibility based on B_a and $B2_a$, and subsequently incorporating RB and $RB2$ in the computation of entitlement B_g . We see that (granting the probability that a unit mistakenly reports a retirement pension instead of its receipt of SB and taking expected values)

expected eligibility -- both in number and in average benefit entitlement -- increases somewhat for pensioners, as was anticipated, along with a large increase in average benefit entitlement. The last column shows the size of the final and complete sets of the eligible, with a rise of more than 100 units compared to what was suggested by the use of calculated B_a . The bulk of that increase comes from the effect on the subsample of pensioners, whose entitlement we have estimated to be sizeably underestimated by B_a . Such reassessments of entitlement are, by themselves, expected to have an important impact on the rate of take-up, as can be checked on Figure 4.4. Using fully revised entitlement figures pushes down and right to B the take-up curve A based on calculated eligibility and on observed FES claims. Curves C and D display on the left vertical axis the average entitlement figures for each of the centiles of calculated entitlement of the horizontal axis, with revised entitlement D lying fairly close to calculated entitlement C.

We must not forget that one driving force behind the increase in projected entitlement is the failure of some pensioners to report accurately their SB receipt. In Figure 4.5, curve A plots observed FES claims from pensioners, whereas B shows predicted FES declarations of claims and C displays predicted claims by pensioners. Comparing A and B indicates that our predicted results fail to fill a fairly large accumulation of declared claims starting at revised entitlement $B_a + \mu_e$ (allowing for the probability that $B2_a$ ought to replace B_a) of about £5, and suggests that a more refined model of pensioners' SB reports might be needed, allowing, say, for a lesser likelihood of benefit confusion for those with apparently low entitlement to (and lower needs of) SB, and conversely for the others.

Nevertheless, we must also notice that the overall level of pensioners' reports of SB is well untangled, as the ultimate crossing of curves A and B shows. Finally, allowing for many more actual pensioner recipients of SB than there are, declared, in the FES data will, of course, swell the rate of take-up. Curve C of Figure 4.5 exhibits this **actual** take-up rate predicted by our econometric results for pensioners; it is much higher than that displayed by A, which does not allow for mistaken concealments of SB receipts.

We end this chapter by summarising our take-up results in Table 4.12, where the findings are shown for the whole sample and for the subsamples of pensioners and non-pensioners and where eligibility is computed either on the basis of B_a or on that of $B_g=B_a+\epsilon$. The column "claim" shows that registered FES receipts of SB are closely interpreted by the econometric estimates and that expected actual claims are predicted to differ significantly from the declared ones by an amount of $645.0-518.9=126.1$. Adding eligibility figures to these results permits the computation of take-up rates, which are shown on the two last columns. Because revised entitlement $B_a+\epsilon$ (allowing for the probability that B_{2a} ought to replace B_a) is on average greater than B_a , the last column shows estimates which are smaller than those based on B_a . Allowing for benefit confusion almost doubles the final estimate of pensioner take-up, increasing it from 43.8% to 78.3%, but due to a higher average estimated burden to claiming for that group, this take-up estimate still remains slightly lower than the non-pensioner take-up rate of 81.3% .

Conclusion

This chapter applied the methodology developed in chapter III on furthering our understanding of the take-up of state benefits. We first modelled some of the suspected deficiencies of our 1985 FES sample of British family units and discussed the apparent socio-economic determinants of the take-up of SB. We subsequently presented the results of our estimation procedure. Among other findings, we note that, for our sample, random and systematic divergences between the calculation of entitlements by the government agency and by the analyst are substantial, and that ignoring them will lead to misleading positive and normative observations. A number of variables are estimated to have a statistically significant impact on the burden to seeking Supplementary Benefits, *viz*, the number of adults, absence from work, the importance of the income unit within the household, and the mean wage and the level of pre-SB gross income. Estimated costs to claiming vary greatly across types of units. Under some stronger assumptions, it is also possible to illustrate the extent to which administrative errors on the part of the government agency and costs to claiming may misallocate (or, in the case of costs to claiming, may sometimes help to allocate) state support among the population. We have also seen how the unsuspected presence of "modelling errors" may distort statistics on take-up rates and how an estimation of the level of entitlement as assessed by the government agency can amend our understanding of the take-up of state benefits.

Table 4.1

Calculated Eligibility and Reported Claims of SB
in the FES Sample

Number of family units

	Positive calculated entitlement		Negative calculated entitlement	
	Claim	Not claim	Claim	Not claim
Total	453	253	50	759
Pensioners	128	160	21	326
Self-employed	4	12	0	181
Single Parents	67	6	3	24

Table 4.2

Frequency Distribution of Claimants and Non-Claimants

Calculated Entitlement (Pounds Per Week)																				
Between		<	-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	>	
And		-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80	80	
Clai- mants	AF	0	0	0	0	4	1	7	12	26	128	31	127	67	40	31	23	3	3	
	RF	0	0	0	0	0.8	0.2	1.4	2.4	5.2	25	6.2	25	13	8.0	6.2	4.6	0.6	0.6	
	CD	0	0	0	0	0.8	1.0	2.4	4.8	9.9	35	42	67	80	88	94	99	99	100	
Non- Clai- mants	AF	93	28	32	33	43	69	87	133	241	185	18	34	7	0	5	1	1	2	
	RF	9.2	2.8	3.2	3.3	4.2	6.8	8.6	13	24	18	1.8	3.4	.7	0	0.5	0.1	0.1	0.2	
	CD	9.2	12	15	18	23	29	38	51	75	93	95	98	99	99	100	100	100	100	
Take- up ¹												41	63	79	91	100	86	96	75	60

AF: Absolute frequencies

RF: Frequencies relative to sample of claimants or no-claimants

CD: Cumulative distribution at end point of range of entitlement

¹ The rate of take-up (in %) is measured by range of entitlement, using the number of claims divided by the number of calculated eligible units.

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Table 4.3

Population Eligibility and Take-up² in 1985

DSS Data
(Our Initial Data)

	Number of recipient units thousands		% Take up (caseload estimates)
	Receiving	Eligible but not receiving	
Total	4390 (2906)	840 (1702)	84 (63)
Pensioners³	1450 (980)	380 (1023)	79 (49)
Single Parents	520 (332)	20 (26)	96 (93)

² Take-up rate as defined by the DSS: (number of claims) / (number of claims plus number of eligible but not receiving). The source of the DSS data is Department of Social Security (1989).

³ In apparent receipt of a National Insurance pension.

Table 4.4

Amounts of Entitlements and Claims in 1985

DSS Data
(Our Initial Modelled Data)
[Using FES Declared Receipts]

	Average £ per week		Total value of SB £m per annum		% take-up ⁴ (expenditure based estimates)
	claimed ⁵	not claimed	claimed ⁶	not claimed	
Total	25.50 (20.20) [25.21]	12.60 (10.50)	5810 (3053) [3810]	550 (929)	91 (77) [80]
Pensio- ners⁷	7.60 (5.60) [4.17]	4.40 (4.19)	570 (285) [213]	90 (223)	87 (56) [49]
Single Parents	40.20 (27.25) [37.13]	40.80 (29.00)	1090 (470) [641]	40 (39)	96 (92) [94]

⁴ Take-up rate as defined by the DSS: value of benefit receipts over the combined value of receipts and that of calculated benefits apparently not taken up. Source of official DSS data: Department of Social Security (1989).

⁵ Value of benefit claimed (as defined in the second column to the right) divided by the number of claimants of the previous table.

⁶ For our grossed-up data (), this is the amount of calculated benefits for those declaring receipt. For the FES data [], figures show grossed-up declared amount of receipts.

⁷ In apparent receipt of a National Insurance pension.

Table 4.5

Preliminary Sample Take-up Statistics (caseload)

	Eligible	Claim	Group Take-up Rate ⁸ (%)
Total Sample	702	503	72
Single Parents	73	70	96
Couples With Children	73	68	93
Couples	169	131	78
One-Adult Units	537	372	69
Special Housing	426	320	75
Renting, but not Special Housing	61	41	67
Head not Working	617	446	72
Older Units	326	175	54
Recipients of NI Pension	288	149	52
Self-Employed	16	4	25

⁸ Take-up rate calculated as number of claimants divided by number of eligible. This is different from those measures used by the DSS and from those suggested by most previous studies.

Table 4.6**Estimation Results**

Definitions	Variables	Estimates of Parameters (standard errors)	
Cost Variables (X) parameters= α		Original	Normalised
Dependent Children	CH	-0.176 (0.200)	-3.81 (4.32)
Number of adults	AD	1.49 (0.351)	32.2 (7.60)
Mean age of adults	M(AGE, AGES)	-0.0192 (0.00995)	-0.417 (0.215)
Single parents	SING	-0.498 (0.600)	-10.8 (13.0)
Head of household	HEAD	-0.749 (0.372)	-16.2 (8.05)
Mean education level	M(ED, EDS)	-0.0053 (0.0303)	-0.109 (0.656)
Special housing	COUNCIL	0.0905 (0.197)	1.96 (4.26)
Renting, though not COUNCIL	RENT	0.505 (0.305)	10.9 (6.60)
National Insurance Pension Recipient	BPR	0.292 (0.393)	6.32 (8.51)
Square root of weeks absent from work	SQRT(WKAB)	-0.407 (0.172)	-8.81 (3.72)
Head not working	UNO	0.291 (0.334)	6.30 (7.23)
Self-employment	SN	0.589 (0.770)	12.8 (16.7)
Mean hourly wage	M(WAGE, WAGES)	-0.296 (0.108)	-6.41 (2.34)
Log of gross income	L(YGROS)	-0.385 (0.123)	-8.35 (2.66)

Table 4.6 (continued)

Estimation Results

Definitions	Variables	Estimates of Parameters (standard errors)	
Entitlement Indicators ($B_a, B2_a,$ $RB, RB2$), parameters β_1 and β_3		Original	Normalised
Intercept	UNIT	0.196 (0.105)	4.25 (2.27)
Calculated entitlement ⁹	$B_a, B2_a$	0.0462 (0.00403)	1.0 (-)
Self-employed	SN	-1.10 (0.908)	-23.9 (19.7)
Self-employed's gross income	SN*YGROS	-0.0202 (0.0149)	-0.437 (0.323)
Other Variables			
Probability of some pensioners not to report SB	BPROB	0.469 (0.0375)	0.469 (0.0375)
Effect on housing benefit of claiming SB	BHCS	0*	0*
Standard deviation of costs	sin (ξ)	0.465 (0.00411)	9.66 (0.0890)
Standard deviation of benefits	cos (ξ)	0.885 (0.00216)	19.25 (0.0468)
Log-likelihood		-461.92	
Number of observations		1515	

* : The estimate of β_2 is at its lower limit: the non-negativity constraint is binding.

⁹ $B2_a$ complements B_a in the set of entitlement variables of those pensioners who may have confused a SB payment for one of NI basic pension. See the text for more details.

Table 4.7
Comparison of Results with Previous Studies¹⁰
Effects on the Probability of Claiming

	(A)	(B)	(C)	Our estimates	(D)	(E)	(F)
Entitlement	+	+ *	+ *	+ *	+ *	+ *	+ *
Presence of additional children	NA	+	NA	+	+	+	+
Presence of additional adults	+	+	-	- *	+/-	+/-	- *
Age	+ *	-	+	+	-	+/-	+/-
Single parent	NA	+ *	NA	+	+	+/-	+/-
Head of household	+ *	+/-	NA	+ *	NA	NA	NA
Education	NA	-	+	+	-	- *	+
Private tenants	+ *	-	-	-	- *	-	- *
Owner-Occupiers	- *	-	-	-	-	+/-	+
Short-Term Unemployed	NA	- *	NA	NA	NA	NA	NA
Household Income	NA	- *	-	+ *	- *	-	+/-
Part-time Work	- *	- *	NA	+	+	NA	NA
Spells Away From Work	NA	NA	NA	+ *	+/-	NA	NA
Long-Term Unemployment	NA	+	NA	NA	NA	NA	NA

(A) Altmann (1981)

(B) Fry and Stark (1987) for non-pensioners

(C) Fry and Stark (1987) for pensioners

(D) Blundell, Fry and Walker (1988)

(E) Dorsett and Heady (1991), on Family Income Support

(F) Dorsett and Heady (1991), on Housing Benefits

¹⁰ Altmann's study covered a sample of male pensioners gathered over several years of FES; Blundell, Fry and Walker analysed the take-up of Housing Benefits (HB), of which the determinants may conceivably be different from those of SB, also studied by Fry and Stark. Dorsett and Heady attempted to estimate the interdependence between the take-up of HB and that of Family Income Support (FIS) between 1984 and 1987. "NA" is indicated when the effect of the characteristic was not part of the study. A "+" (or a "-") shows that the characteristic is found to increase (or decrease) the probability of a claim, other things being equal. A "*" manifests statistical significance.

Table 4.8**Expected Costs (£ per week) to Claiming Supplementary Benefits****Variations in Circumstances of 3 Reference Units¹¹**

Changes in Characteristics	Family of 4 AGE=35, AGES=30, CH=2	Single Parent, AGE=30, CH=1	Pensioner, AGE=75, UNO=1, WKAB=0
(reference unit)	18.4	2.97	3.76
+1 child	15.4	2.71	-
AGE => +10	15.2	2.68	3.33
AGE => -10	22.0	3.32	4.30
ED => +6	17.9	2.92	3.69
COUNCIL=1	20.1	3.13	4.00
RENT=1	28.5	4.06	5.47
UNO=1, WKAB=26	3.66	1.45	-
SN=1	30.3	4.31	-
YGROS=50	16.9	2.84	3.56
AD=2, AGES=70	-	-	21.5

¹¹ Initially, all typical units have the following characteristics, besides being home-owners: HEAD=1, WAGE=£4.5, YGROS=£40, ED=16, and, where applicable, WAGES=£2.00 and EDS=16.

Table 4.9

On Type I and II Errors,
Under the Assumption That $\varepsilon \equiv \varepsilon_0$ and $B_0 \equiv B^*$
Adjusted for the Estimated Probability of Benefit Confusion by Pensioners

	Type I errors		Type II errors	
	Potential $B_0 \leq 0 \mid B^* > 0$	Occurrence $NB \leq 0 \mid B^* > 0$	Potential $B_0 > 0 \mid B^* \leq 0$	Occurrence $NB > 0 \mid B^* \leq 0$
Conditional probability of errors over the relevant sample	0.181	0.302	0.188	0.098
Costs of errors in benefit level (as % of total benefit payable by agency)	6.68	13.0	7.55	5.45
Number of mistaken cases (as % of total recipients)	23.2	38.8	20.0	10.4

Table 4.10

Calculating the Rate of Supplementary Benefit Take-up

	A Claim	B Eligi- ble and claim	C Eligi- ble but not claim	D Predic- ted to claim	E Predicted to be eligible	A/ (A+C) %	B/ (B+C) %	A/ (B+C) %	D/E %
Total	503	453	253	645.0	806.7	66.5	64.2	71.2	80.0
Pensioners	149	128	160	286.7	366.4	48.2	44.4	51.7	78.4
Self-employed	4	4	12	5.2	7.9	25.0	25.0	25.0	66.4
Single Parents	70	67	6	68.9	72.1	92.1	91.8	95.6	95.5

Table 4.11

On Entitlement and Eligibility

number eligible to claim SB
(average benefit entitlement)

	Basis of eligibility calculations		
	Calculated entitlement, with B _a	Expected calculated eligibility, with B _a and B2 _a	Revised eligibility with RB and RB2
Total	706 (20.61)	758.21 (29.15)	806.73 (30.75)
Self-employed	16 (38.25)	23.09 (32.44)	7.86 (25.95)
Pensioners	288 (5.19)	329.39 (23.70)	366.40 (25.29)
Single parents	73 (29.89)	70.24 (32.62)	72.10 (34.76)

Table 4.12

The Take-up of Supplementary Benefits

Groups and basis of computation		Claim	% Take-up ¹²	% Take-up ¹³
Total	as declared in FES sample	503	71.2	62.3
	expected to declare in survey	518.9	73.5	64.3
	expected SB claim	645.0	91.4	79.9
Pensioners	as declared in FES sample	149	51.7	40.7
	expected to declare in survey	160.6	55.8	43.8
	expected SB claim	286.7	99.7	78.3
All but pensioners	as declared in FES sample	354	84.7	80.4
	expected SB claim	358.3	85.6	81.3

¹² Basis of entitlement is calculated B_a .

¹³ Basis of entitlement is $B_a + \epsilon$, also allowing for $B2_a$ and $RB2$.

Figure 4.1

Entitlement and Distribution of Pensioners' Claims

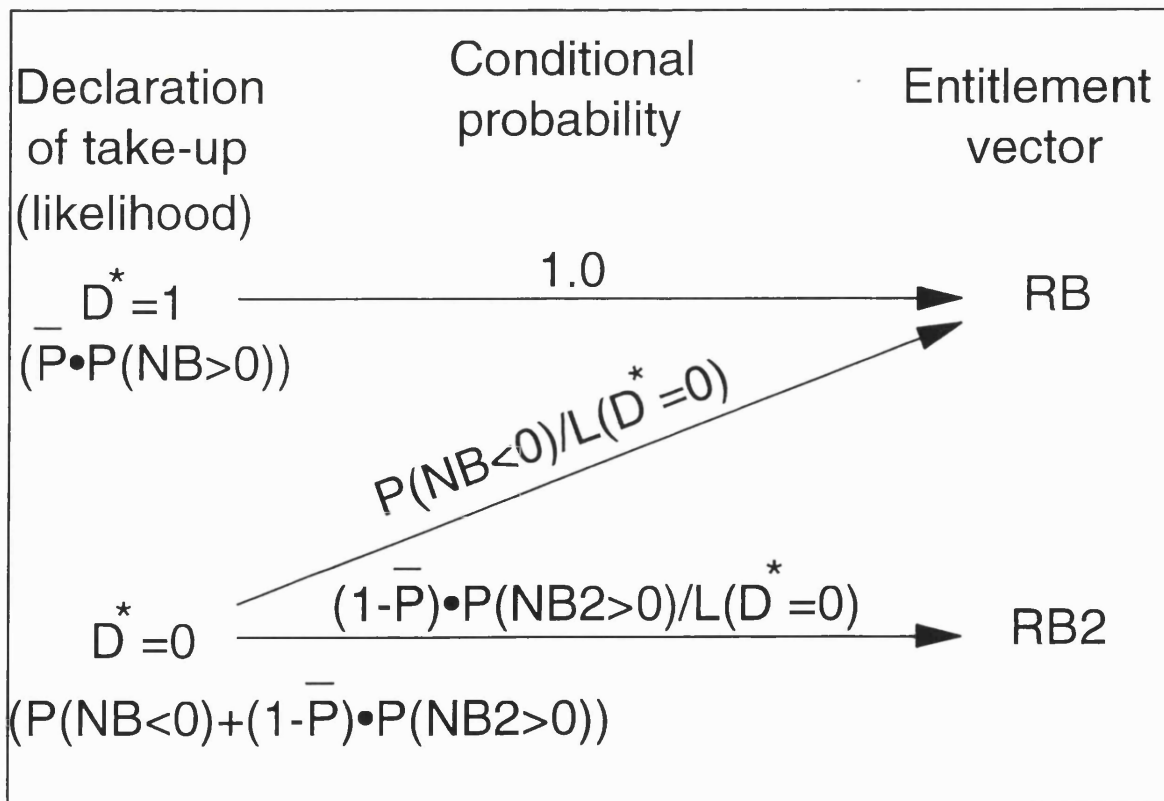
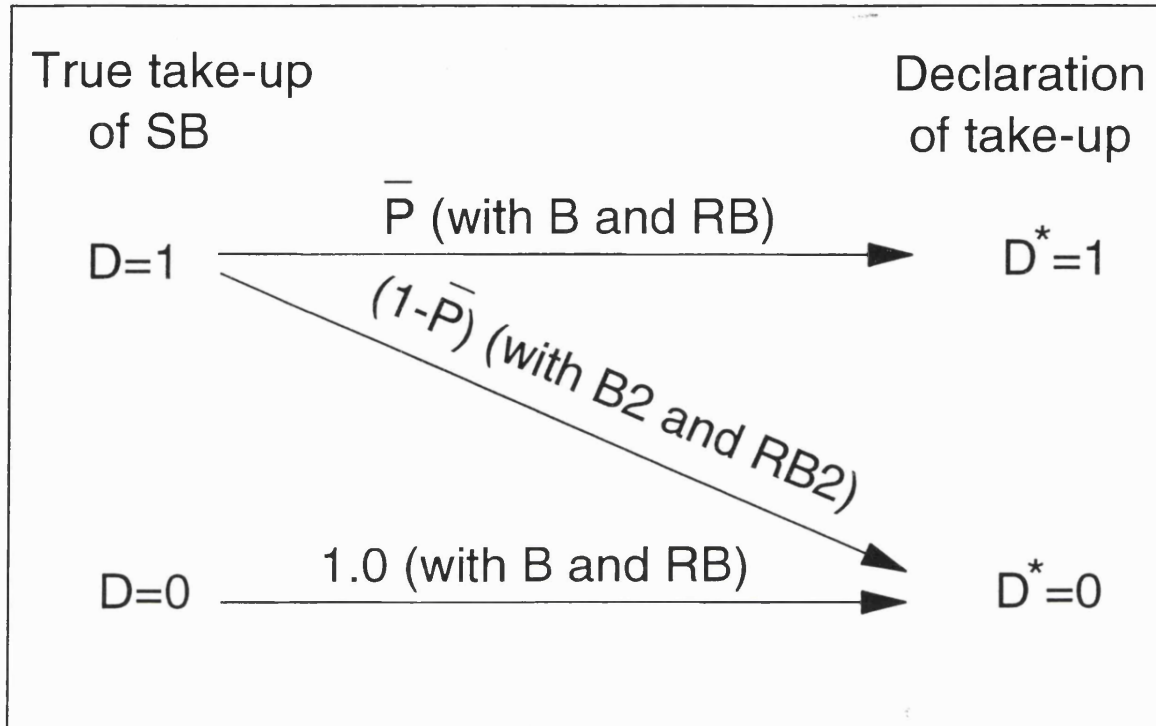


Figure 4.2

Cost, Entitlement and Probability
of Claiming Supplementary Benefits

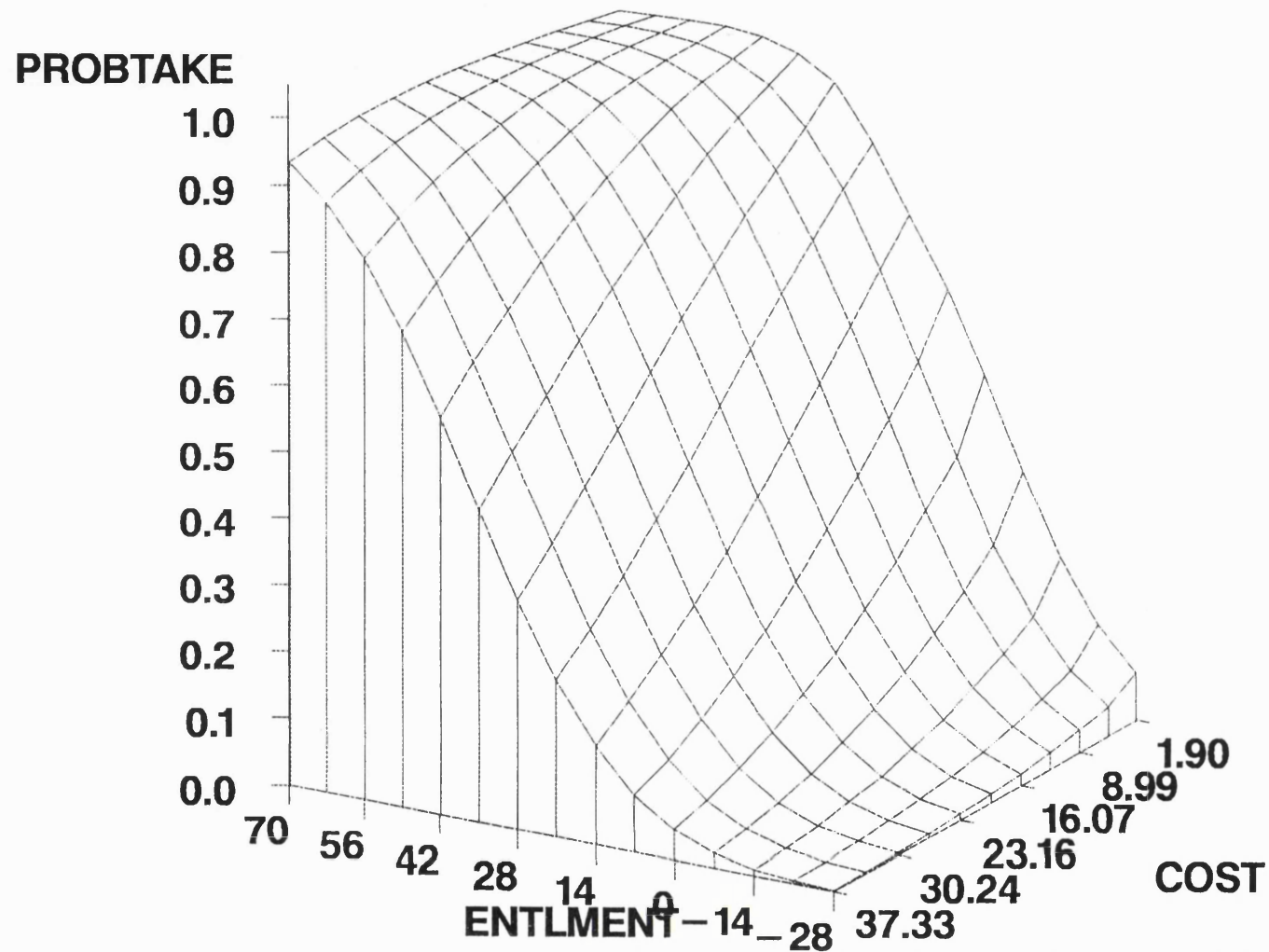
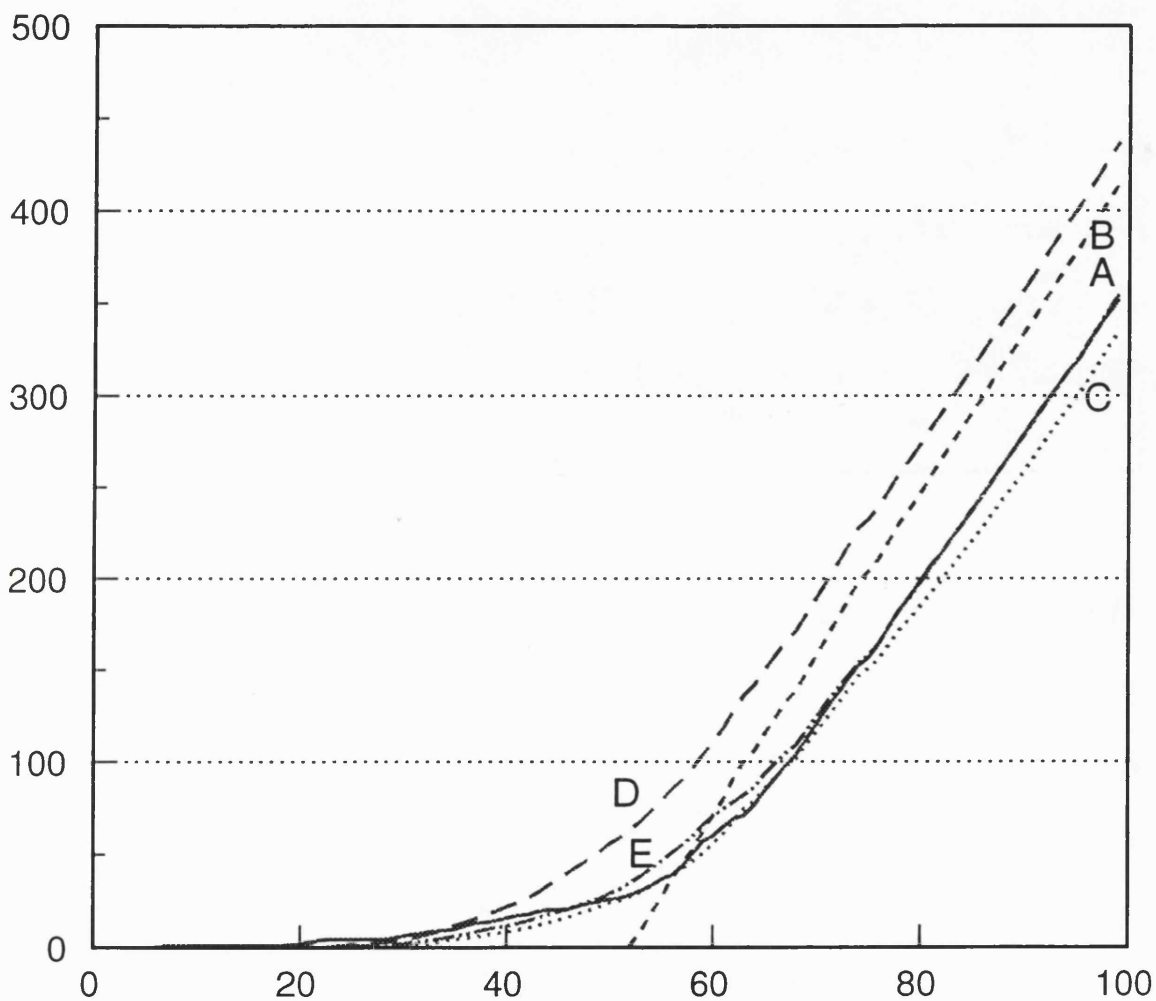


Figure 4.3a

Fit of our Model

Excluding Pensioners

Number of Claims



Centiles of Calculated Entitlement

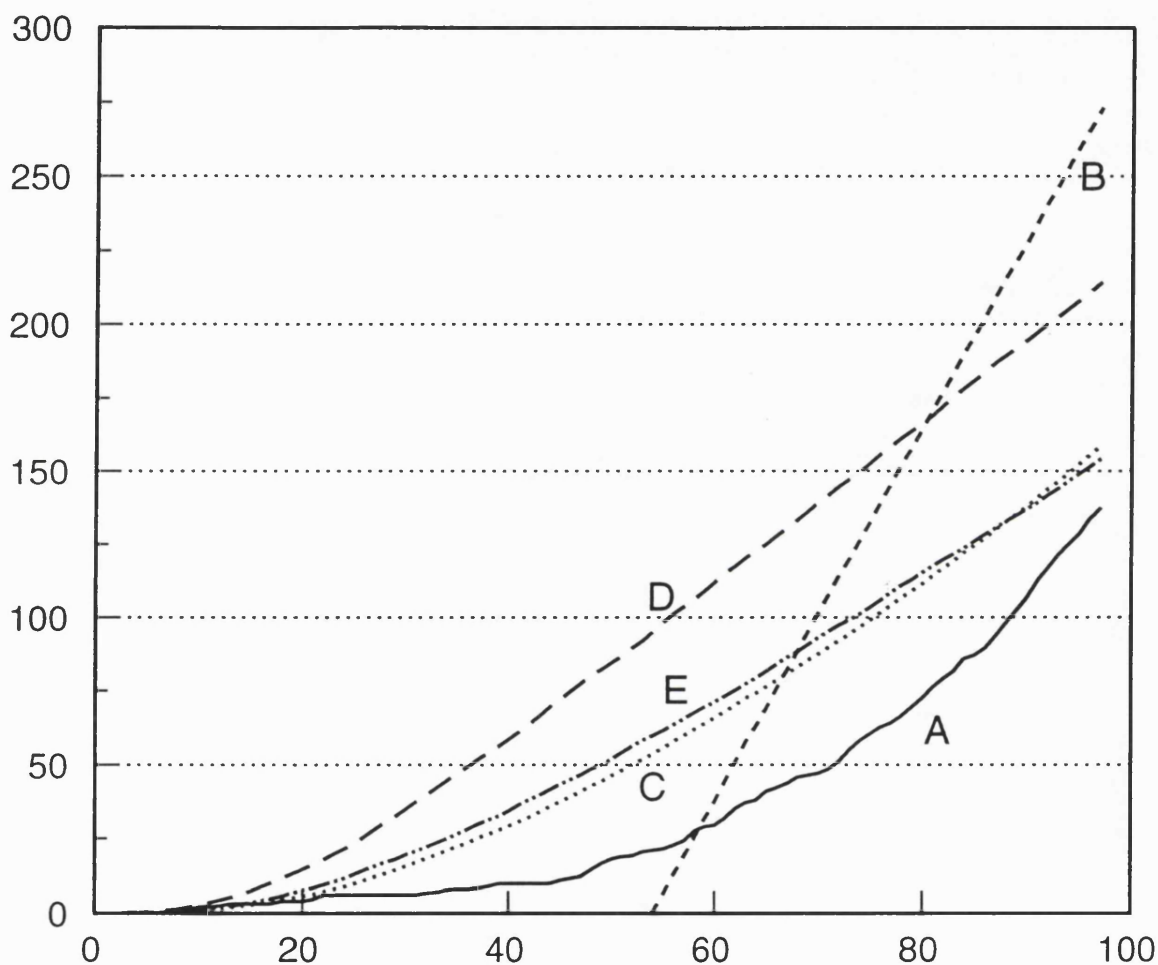
A Observed claims
 B Calculated entitlement No cost to claiming
 C Univariate Probit, No cost to claiming
 D Our full model No cost to claiming
 E Our full model With cost to claiming

Figure 4.3b

Fit of our Model

Pensioners

Number of Claims



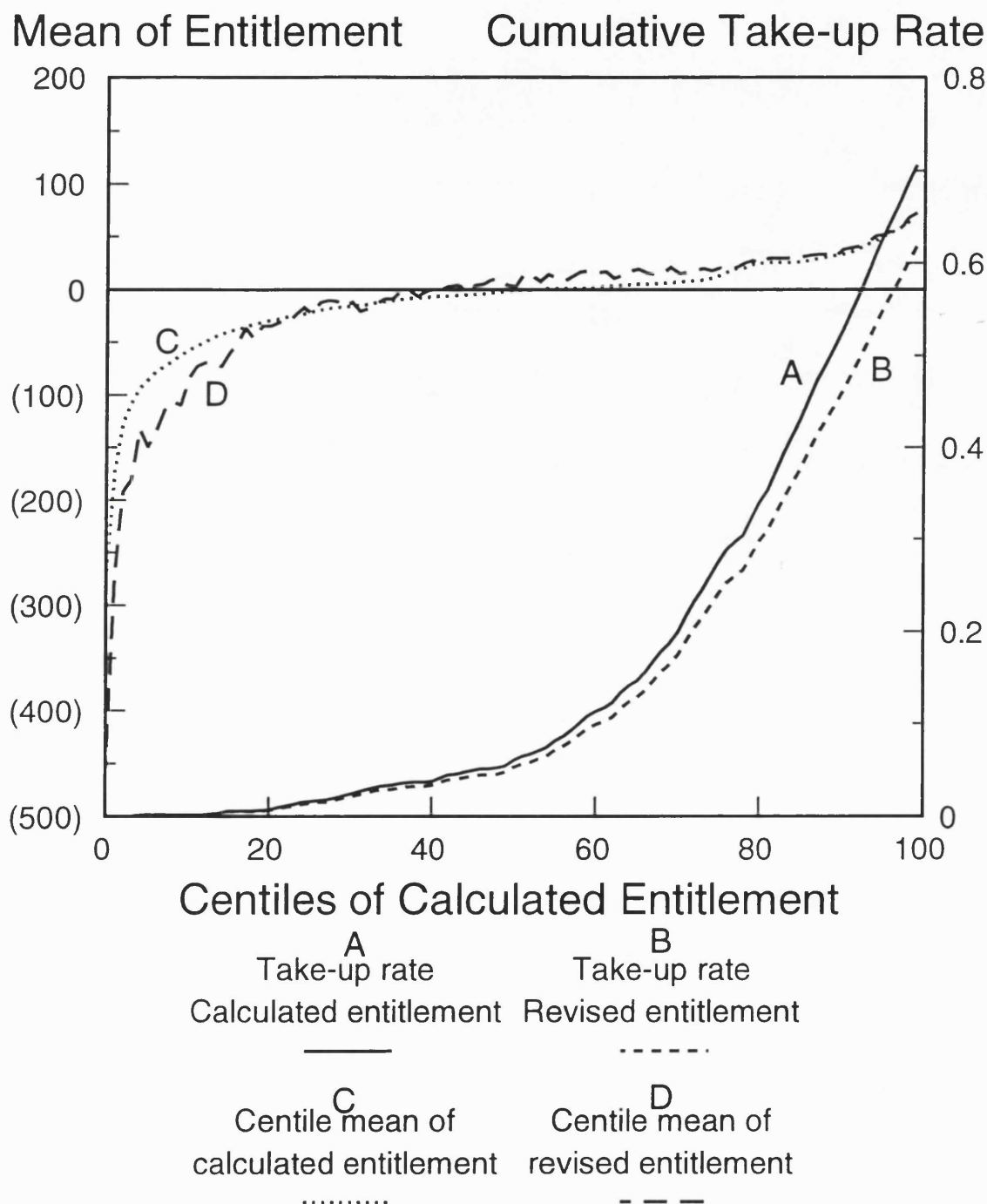
Centiles of Calculated Entitlement

A Observed claims
 B Calculated entitlement No cost to claiming
 C Univariate Probit, No cost to claiming
 D Our full model No cost to claiming
 E Our full model With cost to claiming

Curves under the full model allow for benefit confusion and misreporting.

Figure 4.4

Entitlement and the Rate of Take-up Using Observed FES Claims

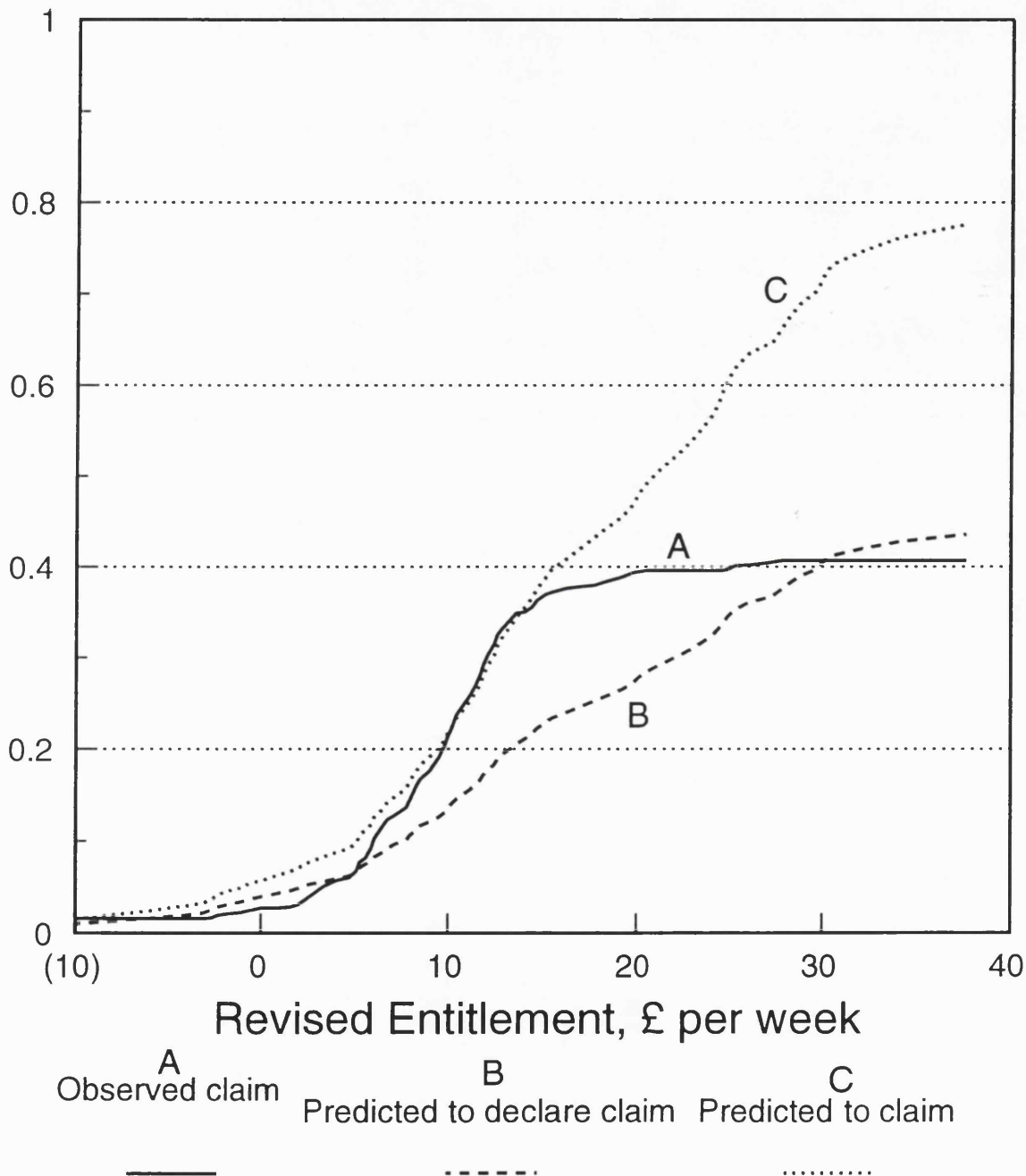


Entitlement is either as originally calculated
or as revised (with revision based on the econo-
metric results and including B2 and RB2)

Figure 4.5

Misreport of SB and the Rate of Take-up Data from Pensioners

Cumulative take-up rate



Entitlement figures are established using the appropriate weighted averages of (B,RB) and (B2,RB2)

Chapter V: Income Support, Contracting Costs and

Social Welfare

Introduction

Our enquiry into the burden of claiming state benefits has made it possible to attempt to understand the behaviour of eligible claimants and their decision to avail themselves or not of state support. It has also enabled us to throw light on the presence of discrepancies between our assessment of entitlement and that we estimate would be carried out by the DSS. In this chapter, we will consider the theoretical and empirical effect of such imperfections in the administration of state benefits upon the optimal allocation of benefits and on social welfare and equity. That is, we go beyond the typical analysis of the effect of state benefits by considering not only what might have been their intended effect -- e.g., granting income support to bring everyone at or above a particular poverty line -- but also their unintended and suboptimal real impact on the members of a given society¹. We shall proceed in several steps.

Firstly, we provide a simple model of the optimal allocation of state support in the absence of uncertainty but in the presence of units which differ by their levels of original income and costs to claiming. The analysis is conceptually different from the traditional study of optimal income taxation [as reviewed, for instance, in Tuomala (1990)], where labour supply and income generation are endogenous but where redistributive imperfections are generally ruled out [see

¹ This impact will not include the deadweight losses which distortions of household behaviour entail, through, say, the effect of taxes and benefits on the choice of labour supply and savings.

Stern (1982) for an exception to this]. We shall see that some simple rules which hold when income redistribution and poverty alleviation are costless do not hold anymore when more general cases are considered. Secondly, we throw some empirical light on the efficiency of government support in providing net benefits and thus in influencing the level of "social welfare". In the third section, we simulate the effect of changing marginally the administration of Supplementary Benefits in 1985 and we discuss briefly a few of the considerations involved in the optimal design of a redistributive policy. The last two sections present both a theoretical and an empirical analysis of the impact of allocative imperfections upon the level of progressivity, vertical equity, horizontal inequity and redistribution operated by income support programmes in Britain and elsewhere. At the end of this chapter, therefore, we shall have had a valuable survey of most of the important features of the optimal and empirical allocation of state support.

A- Social Welfare and Redistributive Costs

In this section we provide an analytical illustration of how government benefits might be allocated to optimise the level of the state's objective function. Bourguignon and Fields (1990), who are concerned with "the measurement of poverty and the implications for anti-poverty policy" (p.409), have already noted some of the redistributive requirements of a state trying to lower some index of poverty in the presence of perfect targeting of poor units and in the absence of administrative and claiming costs. In particular, they consider how the optimal allocation of state support (as a function of pre-transfer incomes) to the poor population changes when different members of the class of poverty measures discussed in Foster et al. (1984) are chosen or when the Sen (1976) index is

adopted. As we shall see more clearly soon, their analysis is a special case of ours.

Perfect targeting is, of course, not generally possible, and Kanbur (1985) analyses how, for the same Foster et al. (1984) class of poverty measures, the allocation of budget expenditures across distinct groups of the population (each containing initially some poor) should be made when there are spillovers of state benefits upon some who are not poor. In such circumstances, the state will want (through an optimal budget allocation) to equalise the marginal welfare returns to increasing budget expenditures in each of the groups; this, however, will not imply that indices of poverty will optimally take the same ultimate values in each of the population groups. Because of this, "a simple rule such as directing more expenditure towards a group with higher poverty is not necessarily optimal" (p.17). Expending aggregate expenditures uniformly over a group that has, for instance, a larger poverty gap than another one but has relatively fewer poor on a headcount basis would lead to relatively more squandering of state expenditures on non-poor units. On a similar note, Kanbur (1987) studies the principles of state support allocation when only broad and coarse redistributive tools (such as the grant of an equal absolute benefit to all) can be used.

When assessing an optimal redistributive scheme, the presence of imperfect information and a distaste of the associated imperfect targeting must naturally be coupled with concerns about the presence of allocative costs -- which would typically increase with any attempt to improve the accuracy of state support allocation. In fact, as Besley (1990) indicates, "imperfect information may also imply that the optimal cost to be incurred in claiming in claiming benefits is not zero" (p.126). The combined presence of both these features -- imperfect targeting

and allocative costs -- is as (and probably even more) important for developing countries as it is for societies with more economically developed economies. As Ravallion (1992) notes,

"Directly targeted poverty alleviation schemes have been widely used in developing countries. Assessing impacts on the poor is an important but difficult analytical problem, recognizing that administrative capabilities typically fall well short of what would be needed for perfect targeting, and so some often subtle but real costs are incurred by both poor and non-poor in participating" (p.97).

Besley and Coate (1992) examine precisely this question of the optimal distribution of participation costs in populations of poor and non-poor for which direct targeting is not possible. They analyse two separate incentive arguments for the optimal imposition of costs and workfare constraints on welfare programme participants: "a *screening* argument that work requirements may serve as a means of targeting transfers and a *deterrent* argument that they may serve as a device to encourage poverty-reducing investments" [Besley and Coate (1992), p.249]. They discuss among other things the various optimal allocative equilibria of work requirements and lump-sum grants among two groups that differ by *ex ante* unobservable ability. One finding is that imposing deadweight losses and constraints on claimants might sometimes be beneficial and so even more beneficial would it be to impose 'productive' workfare (standing in line waiting for benefits, for instance, would then be replaced by a valuable work contribution to the state). Making ability (and the status of poor) endogenous appears to increase even more the desirability of active workfare or cost-generating policies: increasing the burden of participating indeed encourages the generation of poverty-reducing ability.

In building models of optimal income support, there thus appear to be two

main dimensions:

	Perfect targeting	Imperfect targeting
No allocative costs	A1 Bourguignon and Fields	A2 Kanbur
Allocative costs	B1 our analysis	B2 Besley and Coate

Bourguignon and Fields (1990) have focused on the A1 class of models, where targeting is perfect and allocative costs for both the claimants and the administrative agency are null. Kanbur (1985, 1987) considered models based in the A2 class and thus did not ponder the presence of allocative expenses in granting state support. As clearly shown by the Besley and Coate (1992) analysis, and as documented somewhat by the evidence in Grosh (1992) on Latin American programme experience, these two dimensions are neither fully exogenous nor independent: In order to generate a greater degree of targeting perfection, an administrative agency may find optimal, for instance, to increase its operational costs or the costs which agents must incur in claiming a state benefit. The imposition of sizeable deadweight costs or burdensome workfare constraints would, for instance, be "more likely to be the optimal solution if the truly poor represent a small fraction of the target population" [Besley and Coate (1992), p.254]. This is confirmed by the empirical evidence in Ravallion and Datt (1992), where the presence of a high proportion of poor in two Indian villages leads to results suggesting that

"the direct impact on poverty in theses villages arising from [cost-inducing] employment on rural public works scheme is no greater than could be

achieved with a uniform (un-targeted) allocation of the same gross budget across all households" (p.21).

Welfare reforms would thus typically weigh the costs and benefits of moving across A2 and B1, but rarely would it of course be possible to induce both better targeting and a fall in allocative costs for a constant aggregate budget level.

In studying the optimal allocation of state support for the class B2 of models, one should therefore not take the degree of imperfect targeting as exogenous, but rather as a function of the level of administrative and claiming costs to allocating a state benefit. On precisely this, we already noted in Chapter IV that there were empirical social welfare gains to the presence of claiming costs -- they reduced the occurrence of the Type II errors, those by which undeserving units are awarded state support. The presence of both Type I and Type II errors is a clear case of imperfect targeting; when we add to this the empirical presence of allocative costs, we thus naturally find the class B2 of models to be the most realistic one. We leave, however, the study of this more encompassing class of models to some future endeavour.

Our intent for now is -- in contrast with the earlier studies to which we referred above -- to define the optimal allocative policy for the B1 cases, those in which perfect targeting and identification of units is possible but for which redistributive costs exist. As noted above, these costs can be both administrative (directly borne by the state) and incurred by benefit applicants and recipients (such as those estimated in the last chapter). Later, we shall consider a practical example (using the results of the previous chapter) of a way in which the state may determine the optimal respective sizes of administrative **and** claiming (or participation) in the total allocative costs. We shall also comment on how our

results would be affected by the impossibility (for various reasons) of perfect targeting.

We first assume that the redistributational or poverty alleviation objectives of the state are described by a social welfare or objective function of the form:

$$SW = \sum_{i \in S} W(X_i + NB_i, Z) \quad (1)$$

where X_i is the original income of unit i and NB_i , the net benefit to unit i , of the offer of a benefit B_i^* , and where S defines the set of units over which the social welfare function SW is defined. Z is a vector of all relevant policy parameters. As it stands, SW is akin to that used by Atkinson (1987) in attempting to encompass most proposed indices of poverty. Thus, if S is the set of poor people and W is linear, for instance, in $X_i + NB_i$, SW will be a linear transformation of the "poverty gap" once state support has had its impact in the shape of net benefits. In general, however, SW can take a variety of shapes and yield wide classes of poverty or inequality measures. SW might then be seen as an element of a larger social welfare function, an element for the maximisation of which the government allocates a budget \bar{B} , such that

$$\sum_{i \in S} B_i^{**} \leq \bar{B} \quad (2)$$

with B_i^{**} the actual level of benefit received by unit i . B_i^* represents the level of the offer by the state of some benefit to unit i . Thus, if B_i^* is less than C_i , the positive cost to unit i of claiming B_i^* , then the award of the benefit B_i^{**} and the net benefit

NB_i will be null. However, when $B_i^* \geq C_i$, then $B_i^{**} = B_i^*$ and $NB_i = B_i^* - C_i$. Alternatively, if C_i represents a level of administrative costs necessary to grant income support to unit i , B_i^* is the level of expenditures (gross of administrative costs) which the state is willing to spend on unit i , B_i^{**} is the level of gross expenditures actually spent on i , and NB_i is the level of benefit net of administrative costs which eventually reaches the unit. Costs C_i are only incurred when the gross offer of help B_i^* is in fact claimed or granted. The government department responsible for the administration of \bar{B} will thus wish to

$$\begin{aligned}
& \max_{\{B_i^* | i \in S\}} SW = \sum_{i \in S} W(X_i + NB_i, Z) \\
& \text{subject to } \sum_{i \in S} B_i^{**} \leq \bar{B} \\
& \text{with } B_i^{**} = NB_i = 0 \text{ if } B_i^* < C_i, \\
& B_i^{**} = B_i^* \text{ and } NB_i = B_i^* - C_i \text{ if } B_i^* \geq C_i
\end{aligned} \tag{3}$$

NB_i and B_i^{**} are not differentiable with respect to B_i^* at the point at which unit i becomes effectively entitled to and in receipt of state support, *viz*, when $B_i^* = C_i$. This causes no analytical difficulty since, as we shall see, the optimum solutions of B_i^* for this maximisation problem will never have to lie at the corner points $B_i^* = C_i$.

We define λ as the Lagrange multiplier associated to the budget constraint, and W' as the non-negative derivative of W with respect to its first argument. Thus, to maximise SW , the government will wish to ensure that the following condition is met at the optimum values of B_i^* and λ , for each of the units in the set S to be in receipt of state support:

$$W'(X_i+NB_i, Z) \cdot \frac{dNB_i}{dB_i^*} = \lambda \cdot \frac{dB_i^{**}}{dB_i^*} \quad (4)$$

The optimum value of λ will reflect the social welfare opportunity cost of not using government resources for state purposes other than granting benefits B_i^* (such as reducing taxes). We shall discuss later the optimal conditions of receipt (i.e., whether, for unit i , $NB_i > 0$) of state support. For now, we nevertheless note that the allocation of benefit offers B_i^* below C_i will not matter, for then $dNB_i/dB_i^* = dB_i^{**}/dB_i^* = 0$; that is, the offer of gross government benefits has then no welfare impact and no costs since it is not taken up and it is not granted. For those units to which the state will optimally not wish to grant gross support B_i^* in excess of the necessary redistributive costs C_i^* , we thus find the equilibrium condition $W'(X_i+NB_i, Z) \cdot dNB_i/dB_i^* = 0$.

The above equilibrium condition for those to be in receipt of income support assumes that an interior solution for B_i^* then exists for each recipient unit and that, in particular, the functions W are continuously differentiable at each of the optimal value of X_i+NB_i . There are immediate instances in which this is not the case. The adoption as a SW function of the head count measure, for which W takes a constant value until X_i+NB_i reaches a given cut-off level -- say, a poverty line equal to Z_p -- makes W discontinuous. This SW function will force the government to distribute B_i^{**} in such a way as to raise above the given cut-off level Z_p as many as the units in the set as possible. In order to do this, it will grant income support $Z_p - X_i + C_i$ first to that poor unit for which that amount is lowest, then to the poor unit with the second lowest $Z_p - X_i + C_i$, and so on until the

budget \bar{B} has run out.

Considering the poverty gap as the functional form of W makes SW continuous but not continuously differentiable with respect to $X_i + NB_i$. Choosing it as the preferred W function would force the state to choose recipient units in order to maximise the returns in poverty gap reduction per unit of state expenditure. In other words, the state will wish to minimise the aggregate level of claiming and administrative costs incurred for the total level of expenditures \bar{B} . Or said again differently, the state will attempt to fill as much as possible of the poverty gap, avoiding as much as possible in the process the wasteful spill-over of government expenditures as compensation for administrative and taking-up expenses. Because there are fixed costs to allocating government support per claimant, once a desirable benefit recipient has been identified, the state will wish to spend on him as much as is necessary to raise his net income to Z_p . Thus, the government's agency ought to compute an "efficiency" ratio $(Z_p - X_i) / (Z_p - X_i + C_i)$ of full poverty gap reduction over expenditure for each unit i , and grant benefit $B_i^* = Z_p - X_i + C_i$ first to that unit with the greatest "efficiency" ratio, then to that with the second highest ratio, etc., until the budget \bar{B} is depleted². Because some income assistance to some relatively poor units may yet arouse relatively high claiming costs -- or necessitate relatively high administrative expenses -- the state may thus find it preferable to grant income support to relatively richer units. We

² If the choice of \bar{B} is made on the basis of an exogenous λ (i.e., given a constant level of opportunity cost of government expenditures), the state will generally find it optimal to grant units in S either the full support necessary to bring them to Z_p , or no support at all. In the event, however, in which there were some leftover \bar{B} that was not sufficient to raise **any** one of the remaining units up to Z_p , the government would then need to compare carefully the possible average poverty gap reduction per unit of expenditures for each of the remaining units.

shall discuss more thoroughly below the optimal conditions of poverty alleviation under the choice of the poverty gap as the SW objective function.

Whether a unit derives any net benefit from the offers of B_i^* depends on its original income X_i and upon the redistributive cost C_i of granting it state support. Figure 5.1 illustrates this dependency. The straight line $\lambda \cdot B_i^*$ displays the opportunity cost in social welfare of granting to unit i a benefit B_i^* . The claim of such a benefit to unit i will, however, bring a net benefit NB_i that will increase W_i (the contribution of i to the "social welfare function" SW) upwards from 0, the normalised starting position, once the claiming burden has been redressed. The shape of W_i above 0 will depend on the concavity of the function $W(X_i + NB_i, Z)$ with respect to its first argument and on the original income X_i . Units for which it is possible to find a level of expenditure B_i^{**} for which $W(X_i + B_i^{**} - C_i, Z) - W(X_i, Z) \geq \lambda B_i^{**}$ will be conceded eligibility. Whether W_i eventually reaches $\lambda \cdot B_i^*$ -- and whether, therefore, the social welfare benefit of granting state support to unit i is worth its opportunity cost -- will thus also hinge on the size of C_i , the size of the claiming and administrative costs that limit the appeal and the welfare impact of gross expenditures B_i^{**} .

We show on Figure 5.1 four examples of units. Unit 1, which faces expenses C_1 , will happily be offered and claim benefit B_1^* , with a net benefit reward of $NB_1 = B_1^* - C_1$. The SW improvement produced by such a grant, measures by W_1 at B_1^* , greatly exceeds the opportunity cost λB_1^* of spending B_1^* . Unit 2, which faces the same inconvenience to claiming but has a higher original income, will barely be deemed to be eligible, just as is the case regarding unit 3 with a lower X but a much higher C . Once qualified and claiming, however, the two units 2 and 3

will receive what may be a sizeable net and gross benefit, thus showing an important discontinuity in the allocative function of optimal state support. From the above optimality condition, we may indeed note that, when B_i^* is claimed, the corresponding net benefit shall equalise the post-benefit claimants' income, net of claiming costs; that is, at the optimum:

$$X_1 + NB_1 = X_2 + NB_2 = X_3 + NB_3 = e \quad (5)$$

where e is that value of net income which all eligible units will enjoy (equal, as seen above, to Z_p with the use of the headcount and poverty gap measures as W). Unit 4, which enjoys a relatively large X and moreover faces high costs, does not benefit from the programme. The shape of the curves W_i of the social welfare contribution of net benefits NB_i thus determines if unit i is to be deemed worthy of the state's income support. For instance, all those units with:

- original income no less than X_2 and costs greater than C_2 ,
- or original income greater than X_2 and costs no less than C_2 ,
- or original income no less than X_3 and costs greater than C_3 ,
- or original income greater than X_3 and costs no less than C_3

ought not to be allocated state support. From this, we may already see that the greater the trouble to claiming, C_i , the less the chance of receiving a positive B_i^* but the greater this B_i^* is if the benefit is claimed; furthermore, the greater its original income X_i , the less likely is the unit to take up a positive B_i^* and the smaller is B_i^* if it is claimed.

One could extend this exercise to a whole population of functions W curves and determine the set of (C_i, X_i) for which a benefit would be payable by the state.

The shape of this set would, of course, depend on the nature of the SW function adopted. We illustrate this using two examples.

1- The Poverty Gap

The adoption of the poverty gap measure as an index of SW implies the use of $W_i = -[Z_p - (X_i + NB_i)]$ as a functional form for $W(X_i + NB_i, Z)$. A unit i would then be granted eligibility to state support if the absolute change in the poverty gap from a receipt of B_i^{**} did not fall below the opportunity cost of B_i^{**} . For all such eligible units, the state will also wish to raise their net income to the poverty line, Z_p , with gross benefits or expenditures equal to $B_i^{**} = Z_p - X_i + C_i$. Hence, units will be deemed eligible to state support if and only if:

$$B_i^{**} - C_i \geq \lambda B_i^{**}, \text{ with } B_i^{**} = Z_p - X_i + C_i \quad (6)$$

We can think of λ here as the value attached to reducing the government budget revenue requirement relative to using the money to reduce the poverty gap, with the numéraire being a £1 reduction in the poverty gap. A person or a society may feel, for instance, that a £1 reduction in the poverty gap is worth a £2 increase in tax raised, with a consequent welfare value of λ of $\lambda=1/2$. Thus, for units to be eligible, the welfare benefit of poverty gap reduction must exceed the opportunity cost (in welfare units) of gross state expenditures on unit i . Otherwise, state expenditures would be better spent elsewhere than on income support, or taxes could be beneficially cut. Rearranging the above expression, we equivalently find:

$$B_i^{**} \geq C_i + \lambda B_i^{**} \quad (7)$$

(gross expenditures ≥ allocative cost + opportunity cost)

where, again, $B_i^{**}=Z_p-X_i+C_i$. Allocating income support entails both administrative and claiming costs as well as the opportunity costs of the gross expenditures, and in determining whether unit i should receive such assistance the sum of the costs involved must not surpass the social welfare value of the expenditure. Finally, we can deduce a third equivalent eligibility condition:

$$(1-\lambda) \cdot (Z_p-X_i) \geq \lambda C_i \quad (8)$$

(net welfare value of poverty reduction \geq welfare cost)

This makes the eligibility choice more explicitly dependent on the net value in welfare of a possible state's redistributive intervention. If the welfare value of the fall in the poverty gap (net of the opportunity cost of such a reduction) exceeds the value in welfare of the deadweight allocative cost C_i , eligibility will be granted; if not, the unit ought not be entitled to state assistance.

The definition of an optimal set of claimants can thus be made on the basis of an opportunity cost in welfare, λ , and on the interaction of X_i and C_i . From the above conditions, we see that all units with

$$C_i \leq \frac{(1-\lambda)}{\lambda}(Z_p-X_i) \quad (9)$$

will fulfil the prerequisite to eligibility to income support $B_i^{**}=Z_p-X_i+C_i$. A value of λ equal to 1 would deny eligibility to all units with costs greater than zero; the lower the value of λ and thus the lower the opportunity cost of government expenditures, the easier it is for units to qualify for income support. The condition above thus explicitly defines a (convex) set of the eligible for which the location

of the border is governed by a linear trade-off between C_i and X_i . To know the precise position of such a border and the size of the set of the eligible, we must, of course, stipulate directly or indirectly the perceived value of the opportunity cost of government expenditures, λ . Doing this involves for the state either of three policies:

- set the value of λ directly, taking into consideration the social welfare value of reducing tax revenues relative that of decreasing the poverty gap;
- establish a budget level \bar{B} that reflects the government's political or economic "capacity" to pay and derive the associated value of λ ;
- determine a point (X_i, C_i) that lies on the border of the eligibility set and calculate the associated λ .

We shall here adopt the third option, which combines advantages of greater wieldingness and of easier and more intuitive interpretation. At the border of eligibility, we note that:

$$\lambda = \frac{Z_p - X_i}{C_i + Z_p - X_i} \quad (10)$$

If we were to agree that, for instance, units for which the sum of claiming and administrative costs would equal one-quarter of the poverty line and with original income one-half of the same line (viz, $C_i/Z_p=0.25$ and $X_i/Z_p=0.5$) would barely be deemed to fit in the set of optimal claimants, we would infer an associated $\lambda=2/3$. This indicates that a £2 decrease in the poverty gap is deemed, at the margin, socially worth a £3 increase in taxes. With this information, we can then define the whole set of the eligible. This derived value of $\lambda=2/3$ would tell us, for instance,

that units with no original income at all would yet not enter the optimal set of the eligible if the costs associated with the state's income support were to exceed 50% of the level of the poverty line. All those deemed eligible would, however, be granted government support $B_i^{**}=Z_p-X_i+C_i$ that would equally raise their net income to Z_p .

2- The Square of the Poverty Gap

Our second illustrative functional form for W_i and SW is inspired by a class of measures discussed by Foster et al. (1984) for which

$$W(X_i+NB_i, Z) = -(Z_p - (X_i + NB_i))^a, \text{ with } a > 0 \text{ and } Z_p \geq X_i + NB_i \quad (11)$$

The measure just discussed in the previous section and based on the poverty gap is straightforwardly obtained with $a=1$. We develop here the special case of $a=2$, but (as shall become even clearer) we must stress that the results of our model are general to a wide class of SW functions. Analogously to the first example above, units found to be eligible to state support on this new criterion will be those for which the social welfare gain of government assistance exceeds the opportunity cost of the gross expenditures:

$$(Z_p - X_i)^2 - (Z_p - X_i - B_i^{**} + C_i)^2 \geq \lambda B_i^{**} \quad (12)$$

with $X_i \leq Z_p$

and where, for those eligible and as found above, $B_i^{**}=e-X_i+C_i$, e being that constant value to which the net income of all eligible units will be raised. We can

then develop this algebraic expression, as we did for the case of $a=1$ or for the more simple poverty gap measure above, and find that eligible units will fulfil the prerequisite that:

$$\hat{\lambda} \hat{C}_i \leq -\hat{e}^2 + 2\hat{e} - \hat{\lambda} \hat{e} + \hat{X}_i^2 - 2\hat{X}_i + \hat{\lambda} \hat{X}_i \quad (13)$$

The variables with "hats" (or $\hat{\cdot}$) are those as previously defined but normalised by the poverty gap (e.g., $\hat{e}=e/Z_p$). Hence those units for which the allocative burden of granting income support is sufficiently low will be deemed eligible by the state. Because the return to decreasing the square of the poverty gap decreases as net income approaches Z_p , redistributive policy will only be applied to unit i if $\lambda < 2 \cdot (Z_p - X_i)$, the initial marginal welfare return to raising the income of unit i . If this condition were not satisfied, unit i would not receive income support even in the absence of administrative and claiming costs. Using this feature, it can also be shown that the likelihood of a unit remaining eligible decreases as its original income X_i rises.

Hence, the above equation defines a set of eligible units based on values of original incomes \hat{X}_i and allocative costs \hat{C}_i . Those units with low original incomes or low costs will be granted eligibility; the others will not be considered optimally fitted to receive state support. The value \hat{e} of the constant level of net income which all eligible units will enjoy is intrinsically linked to the opportunity cost of government expenditures $\hat{\lambda}$ by the equilibrium condition that at such level of net income \hat{e} the marginal welfare gain of increasing the value of the state's transfer to an eligible unit is precisely equal to the opportunity cost $\hat{\lambda}$ of such additional expenditure. If such a condition did not hold, the government would

clearly find it optimal to increase its support to the relevant eligible unit (instead of granting assistance to a new, additional unit) since administrative and claiming costs would then already have been "sunk". Hence, it must be that

$$\hat{\lambda} = 2 \cdot (1 - \hat{e}) \quad (14)$$

Using this expression, the border of the eligibility set can now defined by

$$\hat{C}_i \leq \frac{\hat{X}_i^2 - 2\hat{e}\hat{X}_i + \hat{e}^2}{2(1 - \hat{e})} \quad (15)$$

To define the precise set of eligible units in terms of (\hat{X}_i, \hat{C}_i) the state must therefore only determine (directly or indirectly) the parameter value of the opportunity cost of state expenditures, $\hat{\lambda}$. As before, the government may either set $\hat{\lambda}$ directly, agree on a budget \bar{B} , or rule on the location of one of the border points of the eligibility set³. In addition, the state could also decide on the level of \hat{e} at which it feels that the level of all eligible units' net income should be raised.

In adopting such a stance, we explicitly express a value judgement on the limit of acceptability in the use of costly redistributive tools. For a given SW functional form, choosing a border point (or a budget \bar{B} , opportunity cost $\hat{\lambda}$, or equalised net incomes \hat{e}) defines the limiting edge passed which we do not feel

³ Presumably, by setting the location of two of the border points or by combining information from two of the options opened to the government (e.g., setting \bar{B} and finding a border point) the state could also derive the associated values of the parameters $\hat{\lambda}$ and \hat{e} -- the parameter of the Foster et al. poverty aggregate which we take to be equal to 2 in this example.

that there exists any net social welfare gain to income redistribution. This has repercussions not only in establishing one of the limiting cases of redistributive policy, but also in settling the whole set of the poor units which will be in receipt of the state's support and in setting the point of equal net income to which we would like to raise the income of some of these poor units of a society.

We illustrate the policy implications involved in such a choice by using the same border unit as that already introduced⁴: ($\hat{C}_i=0.25$, $\hat{X}_i=0.5$). For such a border point we find $\hat{e}=0.809$ and $\hat{\lambda}=0.382$. Using the inequation above defining the eligibility set, we note that when $\hat{X}_i=0$, for instance, allocative costs can go up to $\hat{C}_i=1.71$ of the poverty line before eligibility to income support ought to be withdrawn. This high deadweight value of \hat{C} is nevertheless consistent with the achievement of the social welfare objective of the state as embodied here by the minimisation of the square of the poverty gap. For all eligible units, net income will be raised to $\hat{e}=0.809$ with state expenditure being equal to $\hat{B}_i^{**}=\hat{e}-\hat{X}_i+\hat{C}_i$. When there are no costs to redistribution, units with original income \hat{X}_i up to the point of \hat{e} will qualify for state support. Figure 5.2 describes such policy implications graphically by showing both the set of the eligible units -- as a function of their original income \hat{X}_i and of the administrative and claiming costs involved in attempting to grant income support to such units -- and, on the vertical axis, the level of expenditures which the government would find optimal to bestow on them for purposes of income redistribution and poverty alleviation. All variables

⁴ We could of course increase the "generosity" of our redistributive and poverty alleviation scheme by increasing the units' level of costs or of original income for which we would still find it acceptable to grant income support. This would correspond to a lower valuation of the welfare opportunity cost of government expenditures and to a greater value of \hat{e} .

are normalised by the poverty line Z_p , and the definition of the eligibility set is that derived in this section for a SW based on the square of the poverty gap.

3- Lessons of the Theoretical Model on the Optimal Redistributive Policy

There are important lessons to be retained from such a figure and from our general analysis. First and as said above, entitlement and expenditure rules similar to those portrayed on Figure 5.2 can be derived for the whole class of SW measures. In this study, we have made them explicit only for the headcount, the poverty gap and the square of the poverty gap measures. As said above, this class of SW measures can also encompass indices of redistributive desirability that incorporate units that would not necessarily be classed as absolutely or relatively poor. Second, state expenditures B_i^{**} (or B on Figure 5.2) for those eligible optimally grow in order to compensate fully for lower original income or higher allocative expenses. Two eligible units, for example, with equal respective differences between allocative costs and original incomes will be granted the same level of state expenditures and be raised to the same level of net incomes.

Third, the case of no claiming and administrative burden is a special case of our investigation: On Figure 5.2, it shows along the line where $C=0$. In the presence of allocative costs, some well-known and intuitive results do not hold any more. It is not true, for instance, that the state is indifferent as to the identity of the poor recipients of assistance aimed at decreasing the poverty gap; as we saw above, there are clear directions on who among the poor should be relieved for poverty. Similarly, we notice on Figure 5.2 that all units with zero costs are optimally eligible to state support regardless of their own resources. With positive or growing costs, eligibility quickly becomes restricted to the very poor. As the

burden of assisting units rises, the marginal welfare gain of supporting those with relatively high original incomes rapidly falls below the opportunity cost of not spending the state budget on other units or for other purposes. Hence, as long as there prevail at least some administrative or taking up expenses in the allocation of state support, not all income units should be raised to the same final net income, but an optimal selection would rather need to be made on the basis of original income and levels of allocative expenses. This result does not require the existence of varying allocative burdens across units: Although the more positive the empirical correlation between levels of original income and redistributive costs, the greater the chance that poor units would be deemed entitled to state support, we only require that there exists at least **some** burden to reaching the better off poor for the richer of them not to feature in the optimal set of claimants. This can be seen on Figure 5.2 for those units with \hat{X}_i at or slightly below 0.8, which swiftly become suddenly ineligible to state assistance with small increases in the level of the redistributive costs necessary to support them. This non-convexity of the optimal level of state expenditures (or entitlement in the absence of administrative burdens) as a function of original income also occurred, for instance, in SW measures that are discontinuous even with $\hat{C}_i=0$ -- such as a SW base on a poverty headcount. With this latter index, with zero allocative costs and for a limited budget \bar{B} , the state might find optimal to raise out of the poverty set only those units with original incomes above a particular floor level. The presence of allocative costs generalises the presence of such non-convexities to the whole class of SW functions.

Fourth, our theoretical model suggests features of optimal redistribution

that are somewhat disturbing, at least when considered in the context of our discussion in earlier chapters. We have noted earlier that, on account of varying allocative costs to reaching units, some relatively richer ones might be deemed optimally eligible to income support whereas some poorer units might be denied it. This result clearly casts doubt on the welfare attributes of the measures of vertical equity introduced in Chapter II. Supporting the poorer and not the richer may generate a greater level of vertical equity and of redistribution but is not necessarily optimal if units differ in ways (other than their original income) that are relevant to the redistributive effectiveness of the state -- such as the presence of varying allocative costs.

As importantly, our theoretical findings here on an optimal redistributive policy of the state also extend an extra shadow on the social welfare bearing of our measures of horizontal inequity. On Figure 5.2, we note that all eligible units will receive income support $\hat{B}_i^{**} = 0.809 - \hat{X}_i + \hat{C}_i$ that will raise their net income to 0.809, also the level of expenditures \hat{B} disbursed on units with $\hat{X}=0$ and $\hat{C}=0$ (the vertical height at the point to the farthest right of Figure 5.2). There are, however, many units with $\hat{X} \leq 0.809$ who do not qualify for state support and whose net income will consequently have to lie below that level. Once optimal state redistribution has taken place, therefore, some of the originally poorer units will enjoy a level of net income above that of formerly richer units.

This reranking is especially likely for those richer units presenting high levels of allocative costs; furthermore, it will also occur among those relatively rich and poor units generating identical allocative expenses. Even more significantly and as can additionally be seen from Figure 5.2, there are originally

richer units with an associated allocative burden lesser than that of units originally poorer that will be denied eligibility and end up relatively worse off than their initially poorer counterparts. This is explained by the feature that in allocating optimal income support, the state ought to choose the recipient units among those for which the net social welfare gains are the greatest; once the redistributive expenses have been incurred, the government will spend on the eligible units as much as is necessary to raise these formerly poorer units to a relatively high net income level. If deemed to be socially important, the explicit consideration of horizontal inequity as a social evil would thus necessarily restrain the effectiveness of state redistribution as measured by the class of SW measures.

B- Policy Effectiveness: Costs, Benefits, and Net Benefits

Costs to claiming act in the above model as a deadweight loss which the government will attempt to avoid by concentrating its offers of help onto those units with lower original incomes (to the extent that W exhibits such an inclination) and lower costs to claiming. Only when greater equality of $X_i + NB_i$ is desired among the relevant population would the state be willing to widen the set of eligible units and to dissipate more funds as implicit compensation for higher claiming inconveniences. As a rule, and ignoring the competing issue of fairness and greater equality, it is thus also more efficient to concentrate additional help on those already claiming, for whom no more resources are squandered on paying for the fixed burden of taking up, thus providing an argument against enlarging the set of units eligible to state support and decreasing it whenever possible, for the same overall budget \bar{B} .

Needless to say, trying in practice to allocate benefits on the basis of the

size of at least partially unobservable and unidentifiable take-up costs is rather difficult, and may generate a considerable degree of perceived and real inequity. Because the size of administrative and (in particular) claiming costs is often difficult to assess and cannot therefore be used as a reliable and discerning guide to poverty alleviation and to the redistributive allocation of state support and taxes, we may not allow the C_i of the previous section to feature explicitly in the individual allocative rules derived above. Similarly, it may be that the government is concerned with income rather than with utility or individual welfare. In the present context, the state could leave out the costs of claiming when evaluating the impact of transfers -- because, for instance, it attaches no weight to the time costs of the claimant or to the disutility of effort or stigma.

There may also be political reasons for which an explicit dependence of expenditures on taking up or administrative costs is not permitted (e.g., pressure groups are concerned about the spread of equal net benefits to units with identical original incomes, not about the possibly differing costs in gross expenditures). As Kanbur (1987) notes, "the administrative costs of schemes which attempt even moderate targeting turn out to be excessive, and any scheme which relies on bureaucratic vetting of low-income households on a case-by-case basis is open to corruption and manipulation" (p.72). The government may therefore be politically or practically constrained to derive an expenditure formula, $B(X)$, that depends explicitly only on original income X (assuming we can at least reliably observe it...). The gain of making the explicit formula $B(X)$ implicitly dependent upon the distribution of C would then hinge both on the accuracy of our appraisal of the distribution of administrative and claiming expenses and on the shape of the

conditional density function, $f(C/X)$. The less precise our assessment of the distribution of C or the lesser the empirical dependence of C on X , the lesser the interest of adapting the allocative formula $B(X)$ to take into account the distribution of redistributive costs.

It can also be suggested (Nichols and Zeckhauser (1982)) that inconveniences to claiming play a useful role in separating deserving recipients of state support from "impostors"; this can occur when income (say) and thus correct entitlement cannot be accurately monitored by the DSS, but when the value of the benefit or that of the costs to claiming are nevertheless function of the unobserved correct levels of income and "merit". If the level of inconvenience to the process of claiming a state benefit decreases with the imperfectly observable true level of entitlement, the imposition of deadweight losses will impose greater relative annoyances on the charlatans and can thus succeed in extracting some of the impostors from the process of claiming; that is, "the demeaning qualification tests and tedious administrative procedures involved in many transfer programmes 'may' serve such a sorting function" (Nichols and Zeckhauser (1982, p.376). Or, as Ravallion sets it in the context of developing countries, "work requirements can provide seemingly excellent incentives for self-targeting in that the non-poor rarely want to participate and a great many of the poor do" (p. 102). The additional transfer of resources from the "rich" to the truly poorer that is then possible may well yield social welfare benefits that exceed the costs which the burden of claiming or participating imposes on the remaining state support recipients. This policy would become even more attractive if the imposition of costs on claimants helped diminish expenses of administering the transfer

programmes.

It is thus interesting to consider the extent to which resources might be empirically "dispelled" by existing benefit systems as simple compensation for the cost of taking up state support. Our econometric estimates allow us to do this, and we illustrate the results in £ per week in Table 5.1. The amount of benefits paid appears in the second column, and net benefits flowing from the grant of SB are indicated in the last one. The average level of predicted benefits per claimant is well over £30 per week, except for pensioners. The third column of Table 5.1 displays the level of costs which we would expect, *ex ante*, units with observable cost characteristic $X\alpha$ to face if they claimed SB. The next column shows the level of costs incurred by the same units when they choose to claim. The cost figures of the third and fourth columns differ since those units with unexpectedly (and unobservably) low costs to requesting the state benefit will also be more likely to claim Supplementary Benefits. The discrepancy between "ex ante expected costs" and actual "costs incurred" thus displays the extent to which the decision to take up SB reveals lower than *ex ante* expected costs on the part of the units.

The total net benefit is equal to 82.8% of the total payment of SB; this would suggest that approximately one-fifth of the total income support (SB) budget might be lost to claimants in the form of various inconveniences to claiming. Thus the net benefit of state support to recipients can differ sizeably from the level of benefits offered. As we have seen above, this can have important implications for the desirability and efficacy of transfer programmes. It also ought to influence the labour supply choices of those comparing the utility of full-time work, say, with that as a state support recipient, for which studies have typically

assumed that no cost was attached to the receipt of state support. Average costs per claimant are highest for the self-employed and lowest for the single parents; that, indeed, makes the average claiming self-employed much worse off from claiming SB than an average single parent and slightly less well off than an average pensioner, though a look at the "benefits" column indicates that self-employed units are expected to receive, in gross benefits, on average more than pensioner units. Inferring post-benefit welfare from the level of benefits claimed might therefore be a tricky procedure, even for comparisons of group averages. Contrasting columns 3 and 4 also shows that taking into account the inevitable "self-selection" of units -- namely, the fact that those units with unobservably smaller taking up annoyances will also be more likely to incur the claiming costs - - would decrease the expected burden assumed in claiming by about 15%, relative to the ex ante expected levels.

That approximately only 80% of those entitled do receive SB and that, for these recipients, about 20% of the total benefits paid are lost in deadweight claiming costs clearly raise important issues about the design of redistributive policies. The deadweight cost efficiency of redistributive programmes would be the greatest for those programmes -- *such as child benefits, "basic incomes", "social dividends", or state pensions that are not means-tested* -- that are universal or that depend on characteristics that are easily observable and cannot be readily altered -- *such as age or citizenship*. These forms of state support involve the least physical and psychological burdens to receiving the state's support. Such programmes are also more efficacious since their take-up rates are typically very high, thus preventing the occurrence of sizeable "holes" in the safety net.

In practice, unfortunately, those tools of redistribution that are the most effective in achieving high take-up rates and low claiming ordeal are also the ones more likely to be most costly in aggregate benefits and the ones least "targeted" towards the poor. Besley (1990), who analyses and simulates the alleviation of poverty using the same Foster et al. (1984) class of measures as that employed above, describes the condition as follows:

"Universal provision entails a cost in the form of a leakage of some of the benefit to the non-poor. On the other hand, means-tested programmes may be costly to administer since they require a test of eligibility for claimants. They also impose costs (psychic and pecuniary) on the poor who have to claim, which may deter some of them from claiming" (p.119).

This is conceptually related to the study of Stern (1982), who analyses the extent to which the welfare costs of administrative errors (making, say, Type I and II errors) can outweigh the welfare costs of income taxation (or, in our context, the allocative costs of better targeting). Thus, in alleviating poverty, there exists a trade-off between aggregate benefit expenditures (with their opportunity cost of not making alternative use of them) and the level of efficacy and deadweight cost efficiency exerted by redistributive tools. For instance, a universal and sufficiently high basic income (which may be taxable) would probably alleviate the poverty of a greater number of individuals with a lesser claiming burden than the current means-tested Income Support is able to achieve in Britain, but it would also involve a much greater aggregate expense and consequently higher marginal tax rates for the rest of the population⁵ As seen above, an analogous trade-off also exists, for a given government budget, between targeting precision and

⁵ Marginal tax rates for a full basic income would plausibly hover around 70%. On this, see Parker (1989).

deadweight cost efficiency.

C- On the desirability of marginal reforms to the administration of State Benefits

We can also consider the simulation and desirability of marginal changes in the way the SB scheme is administered. A typical government agency is first and foremost concerned about the level of benefits paid – the sum of which must be, say, kept in proportion with the government's "ability to pay" -- and about its administrative costs, which may well be inversely related to the inconveniences faced by units when applying for an award. As we discussed above, the relevant department should also concern itself about costs and net benefits to claiming, though it will often be more explicitly preoccupied by summary statistics such as the rate of take up. To decide on the best course of action, the department must enter into its objective function U^6 all appropriate information: \bar{B} for the level of total benefits paid, AC for the administrative costs, and NB for the sum of net benefits:

$$U = U(NB, \bar{B}, AC) \quad (16)$$

In choosing whether to change (marginally) its administration of the benefit scheme, the government must ascertain if the modification will, at the margin, improve on the level of U ; that is, for a policy change ΔV in the policy parameter V , whether

⁶ U can be seen as an extended version of the objective function SW used above, where the budget and administrative costs are made endogenous, but where W is reduced to a linear function of net benefit.

$$\frac{\Delta U}{\Delta V} = \frac{\Delta U}{\Delta NB} \frac{\Delta NB}{\Delta V} + \frac{\Delta U}{\Delta \bar{B}} \frac{\Delta \bar{B}}{\Delta V} + \frac{\Delta U}{\Delta AC} \frac{\Delta AC}{\Delta V} \begin{matrix} < \\ = \\ > \end{matrix} 0 \quad (17)$$

Table 5.2 contains the information necessary for the analysis of the marginal desirability of one such alternative. In an effort to cut its budget deficit, say⁷, the government must choose between increasing the costs to claiming benefits (and thus likely reducing the administrative expenses of scrutinising the award of benefits) and decreasing the overall level of entitlement. What possible effects does this alternative subsume?

From Table 5.2 we see that increasing ex ante expected costs by £1.02 will generate a fall in net benefits approximately equal to that of lowering everyone's entitlement by £1. We can therefore ignore the $\Delta NB/\Delta V$ term. As can be anticipated, however, the impact on \bar{B} of the two options is very different: \bar{B} falls when the burden of claiming is increased, for fewer units then bother taking up SB; when entitlement falls, \bar{B} drops yet further from the lesser entitlement of those still claiming. Hence, decreasing entitlement and eligibility allows for additional budget savings of about £669 for our sample, relative to increasing ex ante costs to claiming. If the government department is roughly indifferent between savings of £1 in administrative expenses and identical savings in awarded benefits, then $\Delta U/\Delta \bar{B} \approx \Delta U/\Delta(AC)$ and we need the inconvenience-increasing policy to generate more than £669 in administrative savings relative to the entitlement-reducing one

⁷ A similar analysis could be made for an expansion of the level of budget spending, except for the proviso that there is a limit to the extent to which claiming costs may be decreased without subsequent falls having to be considered explicitly as increases in benefits and entitlements.

to be preferred to it -- that is, additional administrative savings of about £1.06 for each of the 632 units still claiming. These additional savings could not arise much from a reduced application load since the number of expected claimants left in both options is roughly the same. A lesser degree of sensitivity of the department's U to changes in AC would require proportionately greater administrative savings for the inconvenience-increasing policy to be still optimal.

The government, of course, may well be more concerned about more straightforward effects, such as increasing the rate of take-up. In the latter case, it is unambiguously better to decrease entitlement⁸ than to increase costs, since increasing costs leads to roughly the same fall in expected claims but leaves untouched the set of eligible units. The entitlement reducing policy decreases the aggregate take-up rate by only 0.1%, but that rate falls instead by 1.8% to 78.1% when the expected burden to requesting the state benefit is increased.

Finally, the government may be interested in the marginal transformation of additional SB resources into net benefit: the fall in entitlement by £1 removes net benefit at a rate of 83%, basically equal to the average transformation quoted above. Hence, marginal government payments of benefits may or may not be slightly more effective than the average already achieved. This, however, is not entirely surprising. Figure 5.3 shows the costs incurred and the net benefit (NB) derived in an hypothetical sample of units ordered according to their level of entitlement and facing -- for simplicity -- identical and non-stochastic burdens to claiming. Those for whom the benefit offered is no greater than the cost per

⁸ Or not to increase it if the aim is to increase net benefits in an expansionary budget exercise.

income unit of taking up do not claim. Increasing everyone's entitlement by a marginal amount moves the benefit curve up and left and increases the size of the triangle NB and of the rectangle of costs incurred by ΔNB and $\Delta Costs$, respectively. Benefits increase by $\Delta NB + \Delta Costs$: hence, the marginal contribution of benefits to net benefits depends on the size of ΔNB relative to $\Delta Costs$. Marginal claimants do not add to the net benefit of the state support. Thus, we can see that as long as the effect of increasing entitlement on expected claims is not too large, the marginal net benefit of state support will exceed its average level; that, for marginal claimants (and thus for most claimants of a small and limited programme), the net benefit of state support is null; and that, for programmes that are both universal and widely claimed (e.g., child benefit), the marginal net benefit of state support is close to one.

D- Equity, Administrative Errors and Contracting Costs

We can now more easily identify some straightforward reasons for which our earlier applied analysis of the extent of vertical equity and horizontal inequity in 1985 Britain was incomplete:

- (1) the presence of administrative errors on the part of the government agency;
- (2) incomplete take-up of state benefits;
- (3) divergences between the level of benefit and that of net benefit of the state's support to units.

Each of these three imperfections in the operation of the benefit system can limit the magnitude of the redistribution and increase the degree of horizontal inequity exerted by state benefits. We have described in chapter II how one can disaggregate total redistribution -- as indicated by the difference between original

and net income Gini coefficients, $G_X - G_N$ -- into progressivity, average tax (or benefit), vertical equity, and horizontal inequity effects. We quickly recall this discussion in order to identify the theoretical and empirical impact of (1), (2) and (3) on redistribution and equity.

We revise our definition of the Kakwani (1977a) index of progressivity to focus it better on our discussion of the **benefit** system and we omit the subscripts i distinguishing between different benefits. The definition of this revised index (minus the old index) is then given by:

$$\Pi^K = G_X - I_{B,X} \quad (18)$$

$I_{B,X}$ is simply the value of the index of inequality in the distribution of benefit B defined in Chapter II, and G_X is the Gini coefficient of the distribution of original income X . Π^K will be positive for a progressive benefit system: the benefit share of the poorer population will then be at least as great as their share in original income. We can also show⁹ that

$$I_{B,X} = 2 \cdot \text{cov} \left(\frac{B(X)}{\mu_B}, F(X) \right) \quad (19)$$

with μ_B being the average benefit and $F(X)$, the cumulative distribution of X .

To obtain an indicator of vertical equity, we simply multiply Π^K by the average benefit (μ_B) as a proportion of the average net (or post-tax and benefit) income (μ_N). Total redistribution, however, is given by $G_X - G_N$, and $G_N - I_{N,X}$ can then

⁹ This covariance formula is derived, for instance, in Lambert (1989), p.43.

indicate the extent of total horizontal inequity operated by the combination of all taxes and benefits.

We may then discuss the impact of each of the three above imperfections on the progressivity, vertical equity, horizontal inequity and redistribution of state benefits.

1- The Presence of Administrative Errors on the Part of the Government Agency.

These errors may come in two shapes. The government agency responsible for the administration of some state benefit may first systematically over- or underestimate the entitlement of all units. Alternatively, these errors can be a function of observable characteristics (such as part-time self-employment) or may be distributed randomly across the population of units.

Systematic errors in assigning entitlement will affect the equity of state benefits in a relatively straightforward manner. Reducing everyone's benefit entitlement by a given absolute amount will, for instance, increase the intrinsic progressive bite of a typical state benefit since that will concentrate the benefit expenditures on those at the lower end of the income distribution. That systematic fall in benefit entitlement will, however, cause a drop in the average rate of a means-tested benefit sufficient to decrease the level of vertical equity exerted by the benefit. The redistributive impact of the means-tested benefit will also fall, and horizontal inequity should also drop slightly due to the lesser significance of income redistribution. The reverse case can of course be made for an overestimation of or a rise in everyone's entitlement.

Random administrative errors in the computation of entitlement to a

means-tested benefit can be expected to raise the benefit's redistributive impact, although they will intuitively also swell the occurrence of horizontal inequity. Let's denote by $B^*(X)$ the level of correct entitlement, and by B the level of entitlement as assessed by the government agency. We have

$$B = B^*(X) + \epsilon \quad (20)$$

Benefits payable are equal to

$$B^b(X) = \begin{cases} B^*(X) + \epsilon, & \text{if } \epsilon \geq -B^*(X) \\ 0, & \text{if } \epsilon < -B^*(X) \end{cases} \quad (21)$$

with $\delta B^*(X)/\delta X \leq 0$ and ϵ being a random error term with mean zero.

To assess the bearing of random errors ϵ upon the Kakwani index of progressivity we must consider whether

$$I_{B^b(X),X} = 2 \cdot \text{cov} \left(\frac{B^b(X)}{\mu_{B^b}}, F(X) \right) \quad (22)$$

is lower or larger than the corresponding inequality index in the distribution of the benefit corresponding to entitlement $B^*(X)$. Intuitively, random errors ϵ are on average beneficial to **all** units for, although they will never entail a below zero level of benefit payable (that is, taxes are not payable by those with assessed negative entitlement), they may lead to a sizeable exaggeration of entitlement relative to $B^*(X)$. Those, however, whom we expect to benefit most from random administrative errors are the ones at the limit of eligibility to the state benefit: they are indeed the ones most likely to profit from random largesses of the

government. There are thus two effects on the Kakwani index as considered above. First, the average benefit will increase in size and, second, those gaining most absolutely will not lie at the bottom of the distribution but will be found in the area at which eligibility to the benefit ceases. Hence, random entitlement discrepancies will have an indeterminate global impact on the Kakwani index of progressivity. Nevertheless, random administrative errors can be expected to increase the degree of vertical equity and redistribution exercised by means-tested benefits since they will raise the average benefit level and may lead to a sizeable redistribution in favour of a relatively poor (though not the poorest) subset of the population. As mentioned above, we also expect them to lead to a significant level of horizontal inequity.

2- Incomplete Take-up of State Benefits

There are several ways in which one may wish to consider the effect of less than complete take-up rates of state benefits upon equity and redistribution. The first one is by specifying a new benefit function

$$B^b(X) = \begin{cases} p(X)B^*(X), & \text{if } B^*(X) \geq 0 \\ 0, & \text{if } B^*(X) < 0 \end{cases} \quad (23)$$

where $p(X)$ is a probability function that takes values of between 0 and 1. We may check that if $p'(X) \equiv 0$ and that $p(X) \equiv \bar{p}$ the imperfect take-up of benefits does not change the value of the Kakwani index and that the progressive bite of the state benefit is therefore unaffected – B is simply a proportionately scaled down measure of $B^*(X)$:

$$I_{B^b(X),X} = 2 \cdot \text{cov} \left(\frac{\bar{p}B^b(X)}{\bar{p} \cdot \mu_{B^b}}, F(X) \right) = 2 \cdot \text{cov} \left(\frac{B^b(X)}{\mu_{B^b}}, F(X) \right) \quad (24)$$

Vertical equity then drops by the same proportion as the relative fall in average benefit, and horizontal inequity and redistribution similarly dwindle. We do anticipate, however, that $p'(X) \neq 0$ and that, in particular, $p'(X) \leq 0$, in which case the propensity of units to claim a lesser proportion of their smaller benefit entitlement will enhance the progressivity of the state benefit, since it will tend to redistribute the benefit relatively more towards those with lower X . In other words, a take-up rate that increases with entitlement and decreases with X helps to concentrate further the state's support onto the subset of those with the lowest original incomes. The average benefit will, of course, be lower than when a full take-up rate applies. Hence, notwithstanding the greater progressivity, we expect the level of final redistribution operated in the whole population by the state benefit to drop, even in the case in which $p'(X) \leq 0$ ¹⁰.

There naturally arise in the preceding analysis important issues of *ex ante* and *ex post* equity. *Ex ante*, the above procedure does not generate inequality and horizontal inequity among identical units since all entitled units with income X would expect to receive, on average, benefits of $p(X) \cdot B(X)$. *Ex post*, inequality and horizontal inequity would be increased by the feature that a proportion $p(X)$ of the units with income X would be in receipt of $B(X)$ and that $1-p(X)$ would not.

¹⁰ Decreasing take-up rates could nevertheless conceivably increase income redistribution if benefit eligibility was widespread enough to make those at the top of the distribution fail to claim sizeable benefit amounts.

Hence, a second way in which we can consider the effect of less than complete take-up rates of state benefits is by specifying (0,1) events of a successful benefit claim. Progressivity and vertical equity will not be much affected relative to the first option since our sample units would be allocated, on average, the same benefit. This specification, however, simulates much better the *ex post* stochastic distribution of receipts and non-receipts. It will also allow for a sounder analysis of horizontal inequity: for two units with similar income and relevant characteristics, one may well choose to request the state's support and the other may not, the incongruity in circumstances stemming from a different level of costs to claiming. This stochastic approach is the one we shall be preferring in our empirical application below.

3- Divergences Between the Level of Benefit and That of Net Benefit of the State's Support to Units.

As we noted above, the net profit to units of some state assistance can lie well below that of the size of the gross benefit if physical and psychological obstacles to seeking it are important. The existence of constant costs to claiming would definitely decrease¹¹ the net redistributive impact of the state support since such costs would hit absolutely and relatively more those with the lower *X*, *viz*, those who deserve the benefit more. This is so since -- compared to a full take-up case -- those with a relatively low entitlement to the state benefit and a relatively high *X* can always prefer not to bear the claiming costs by choosing not to seek the benefit: the maximum they will then lose is their relatively low benefit

¹¹ Unless the costs led to a fall in administrative expenses that then fuelled a rise in benefit expenditures.

entitlement. Moreover, those who are relatively high in the income distribution and who are not in any case entitled to the state's support will clearly not suffer from the presence of claiming costs. Those, however, with a sufficiently high entitlement to the benefit will still prefer to seek it if requesting costs arise and will therefore bear the full extent of the claiming burden.

The only rare circumstances in which the occurrence of claiming costs could possibly swell the redistributive impact of state benefits is when costs, not being constant across the population, happen to be particularly low for those potential claimants at the bottom of the income distribution (e.g., for single parents and widows). Costs are, however, slightly more likely to raise the progressivity of state benefits since it measures the relative concentration of net benefits across the population and is not lessened by the fall in the average value of the state's assistance to individuals.

Costs to claiming have an ambiguous effect on the level of horizontal inequity relative to what occurs with the imperfection (2). On the one hand, horizontal inequity tends to fall compared to (2) since, for two units with similar characteristics and original income, the net advantage of the one which chooses to claim when the other does not is reduced by the level of costs which the claiming unit must incur and discount. Hence, claiming burdens tend in this way to limit the horizontal unfairness of unequal assistance to units otherwise similar, similar except for the fact that one finds it worthwhile to seek the benefit and the other does not. On the other hand, however, observably and unobservably different costs to soliciting state support tend to increase the variability of the *net* impact of redistribution across the population. Identical SB grants to similar

individuals may yet result in widely different levels of net benefits. It may be seen as unfair, for instance, that a claimant living far away from his or her Social Security office may face significantly higher transportation and information costs than an otherwise similar claimant who just happens to live next door to the Department's local bureau.

E- Equity and the Take-up of State Benefits in 1985 Britain

How much do the three above imperfections in the operation of the benefit system *actually* affect equity and redistribution? We dedicate this last section to an empirical illustration of how the planned impact of Supplementary Benefits in 1985 Britain might have been distorted by the imperfection of the allocative process. We underline "might" since, to render this illustrative analysis possible, we must make some important simplifications in addition to those made in the derivation in Chapter III of our take-up estimation procedure.

We make throughout the implicit assumption that we measure original income, needs and SB entitlement accurately. In particular, we do not correct for a likely underreporting of self-employment income. Nevertheless, we adjust consistently our results for the likelihood of some benefit confusion on the part of some older people, making adequate alterations to the occurrence of National Insurance Basic Pensions. All random divergences between our computation of entitlement and that official computation which we estimate has (or would have) been made are attributed to random errors made by the DSS, the government agency responsible for the administration of Supplementary Benefits. It is, of course, possible that the reverse assumption be valid, that the DSS measure entitlement with precision, and that all stochastic discrepancies ought to be

attributed to our inability to model entitlement and original income accurately. In these circumstances it would then be plausible that most of the horizontal inequity here imputed to DSS administrative errors would disappear; there would still remain, however, the horizontal inequity stemming from varying contracting costs to claiming and from imperfect take-up rates.

Income is always expressed below in the form of equivalent income, using the equivalence scale implicit in the 1985 Supplementary Benefit scale, excluding from such a scale the element of SB springing from the payment of mortgage interest by owner-occupiers. We use the grossing-up weights derived in Atkinson, Gomulka and Sutherland(1988) and we focus our analysis on individuals, not families or households, assuming that family income is equally divided across members and attaching an initial weight to each family equal to its number of members. When modelling the circumstances of a family unit for which there exists a positive probability that it receives income support in the form of SB, we create two observations for which the separate weights sum to the weight of the original unit. In one of these observations, the unit is not entitled to or does not claim SB but, in the other, it receives the level of grant or net benefit conditional on a unit being in receipt of SB. As discussed above, this mildly stochastic procedure can be expected to yield a reasonably good picture of the wide distribution of SB grant and net benefit in the population.

Figure 5.4 indicates the movement of the Lorenz curve A of original income to concentration curves of various income distributions (B to D, ordered by the level of original income) towards the Lorenz curve E of net and final income. The impact of income support on the level of **net** income is then measured through

the level of SB bestowed on units **net of claiming costs** (as imputed, for instance, in Table 5.1). We note the movement of B away from A operated by the sizeable vertical equity exerted by the combination of all benefits and taxes apart from NI Basic Pensions and SB. We also notice that the wobbly increasing and decreasing slope at the bottom of B suggests the existence of a significant amount of horizontal inequity in the movement of A to B: some units towards the right of B have less income (the concentration curve B there displays a shallower slope) than other units towards the left of B (the concentration curve B there displays a steeper slope). Hence, reranking the curve B units according to their income on B would move some of the initially poorer units to the right, passed other units formally richer. This horizontal inequity is greatly reduced as the impact of NI pensions and revised SB entitlement (B_g , with systematic and random administrative errors) is also felt, leading to the concentration curve D. Because concentration curve D incorporates all expected gross benefits and taxes, it lies closest to an equal income distribution curve. To obtain an accurate picture of actual redistribution we must, however, predict actual claims of SB and subtract from the level of gross Supplementary Benefits the amount of claiming costs incurred and we must subsequently rerank the distribution of individuals according to the size of their final net income. This is shown on curve E, which lies surprisingly close to the dotted curve C that includes all but SB. The combination of SB costs to claiming and the level of total horizontal inequity exercised by all taxes and benefits thus appears to withdraw almost completely the vertical equity impact of the grant of Supplementary Benefits.

Table 5.3 disaggregates more clearly than the above figure the impact of SB

upon progressivity, vertical and horizontal equity, and redistribution. As noted in Chapter II, NI Basic Pensions are somewhat less progressive than SB, their Kakwani index of around 1.0 being about 25% lower than the SB's. This is not very surprising since such pensions were mostly granted independently of the receipt of other incomes, whereas the SB's income support is strongly means-tested. We also note that reported FES figures on NI Basic Pensions can overestimate significantly their redistributive impact, for some older individuals will mistakenly declare a receipt of a state pension instead of that of SB. Our results show that the average NI Basic Pension benefit as a proportion of net income drops from 4.9% to 4.4% when we attempt to correct the FES figures, with a concordant fall in the extent of vertical equity exercised by the benefit.

Lines (c) to (g) of Table 5.3 show the variations in equity effected by SB when we harmonise our entitlement figures to those we estimate would on average be computed by the DSS (d), when we allow for random administrative errors (e), when the take-up rate is incomplete and varies according to observable and unobservable characteristics (f), and when the size of SB must be discounted by the presence of claiming costs (g).

The Kakwani index of progressivity varies very little across these different specifications. The biggest change occurs when random administrative errors are introduced, a feature which creates potential holes in the grant of SB and makes the programme less reliable and less globally progressive, causing a fall of the Kakwani index from 1.318 to 1.301. As discussed above, imperfect take-up rates make the SB programme more progressive, since those with a lesser entitlement are also the ones least likely to seek the state's support. Somewhat surprisingly,

the index also increases slightly when costs to claiming are incorporated, suggesting that such costs happen to be relatively low in our sample for those towards the bottom of the original income distribution.

Because progressivity (or the "potential" vertical equity bite of SB) is so little affected by the movements from (c) to (g), it will be the changes in the average support provided by SB that will govern the shifts in vertical equity. We recall that the computation of entitlement by the DSS appears to be systematically more generous than the one we make, and this accounts for the increase from 2.7% to 3.1% in the average benefit when we shift from (c) to (d), and for the matching increase in vertical equity. It is at least as interesting to note that the presence of random administrative errors will cause a further substantial increase in the aggregate level of SB granted. Total SB payable as a proportion of total net income jumps from less than 3.1% to about 3.5%. This can have important applications for the exercise of tax and benefit analysis and for the consistency of simulated aggregate figures with official ones. *Ceteris paribus*, in the presence of random administrative errors¹², we expect aggregate figures predicted by tax and benefit models to underestimate the level of aggregate payments made by the government, an underestimation which can be substantial if we are to be guided by the results of Table 5.3. Fortunately, however, this bias is reversed if the analyst is rather the one mostly responsible for generating the random entitlement discrepancies. This latter situation would arise, say, if the DSS was able to monitor closely the eligibility of current and potential claimants but with the survey information at the disposal of the analyst being relatively inaccurate. In such

¹² So long as the error term has a zero mean.

circumstances, the analyst's computation would on average grant a positive benefit even to those correctly considered by the DSS not to be eligible as well as a higher than appropriate benefit to those deemed positively entitled by the government agency. Too high a level of benefits would then be predicted by the analyst, relative to the aggregate amount awarded by the then more precise DSS.

The level of benefit granted understandably falls when we move to (f) with an imperfect take-up of SB. The extent of vertical equity then exerted by SB correspondingly falls by about 10% of its peak at (f). The fall is even greater when we add to the fact that not every eligible unit does claim SB the consideration that, for those who do receive it, the level of aggregate net benefits is only 80% of that of the SB paid. Net SB contributes significantly less in vertical equity than either payable SB (e) or taken up SB (f), and even less than our original calculated SB entitlement.

When considering the impact of income support upon the level of horizontal inequity exerted by all taxes and benefits, we ought to keep in mind that SB constitutes only a small element (less than 5% of the sum of the absolute payment of taxes and receipts of benefits that are modelled by our tax and benefit computer programme) of the government's overall redistributive tools. Thus, the increase from 0.018 to 0.021 of the index of total horizontal inequity when we add an element of randomness in the allocation of Supplementary Benefits -- a move from (d) to (e) -- must be seen as an important indicator that a significant degree of inequity may prevail empirically in the operation of redistributive policies. Such horizontal inequity is decreased to 0.019 when claiming costs reduce the net advantage of those who do find it worthwhile to seek the state benefit.

Combining the change in the level of vertical equity exerted by SB and the change in the index of the level of horizontal inequity exercised by all taxes and benefits can account for much of the movement in the Gini coefficient shown in the last column of Table 5.3. The Gini coefficient falls to 0.286 when revised (*d*) rather than calculated (*c*) entitlement is used since SB is then greater as a proportion of overall net income, pushing vertical equity upwards. The subsequent and similar increase in vertical equity when random administrative errors are incorporated is mitigated by a sizeable increase in total horizontal inequity, and the Gini coefficient falls to its overall low, 0.283. Less than perfect take-up rates push the coefficient above the level imputed for (*d*) -- where we ignored random entitlement discrepancies -- as vertical equity and the importance of SB as a redistributive tool drop. Finally, incorporating costs to claiming raises the Gini coefficient to an overall high, and thus decreases redistribution to an overall low. There, progressivity, total net income support, and vertical equity all lie below the level corresponding to calculated SB entitlement (*c*) from which we started our analysis, with horizontal inequity slightly higher than at (*c*) but much decreased from its peak at (*e*). Needless to say, these changes in the value of aggregate indices conceal a much greater diversity in individual circumstances stemming from the incorporation of SB allocative imperfections.

Conclusion

Redistributive instruments may have practical effects far different from those intended by legislators and policy activists. Combining the tools developed and applied in earlier chapters, we are able in this chapter to enquire into the

suboptimal and real impact of suspected imperfections in the administration of state benefits. We first consider what inconveniences to seeking state benefits implied for the optimal allocation of benefits in a model of perfect targeting. Simple rules applicable to the case of no redistributive costs do not apply anymore, and the optimal policies derived have important and disturbing bearing on issues of vertical and horizontal equity. We then estimate that incurred contracting costs to claiming amounted to about 17% of the level of all Supplementary Benefits granted in our 1985 sample. These costs are largest for the largest units and are in general unequally spread across the population, making welfare comparisons based on *benefits* and *net benefits* sometimes yield different results even across groups of family units. Our analysis yields some very relevant input into the selection of optimal redistributive tools, *e.g.*, on choosing universal rather than targeted state support. The study of marginal income support reforms suggests that increasing entitlement would not change sizeably the aggregate take-up rate but that decreasing costs to seeking the state's support could increase it much (although this latter policy would also increase the misallocation of benefits to those not considered to be truly deserving them). Reducing contracting costs can therefore be a rather more efficient way of helping the poor than increasing entitlement to state benefits.

Finally, we illustrate the impact of imperfections in allocating state support upon the level of equity and redistribution in 1985 Britain, using for this purpose the framework developed in Chapter II. The analysis is tentative since, in making it, we ignore the likely presence of analogous imperfections in our computation of income and entitlement. We do, however, show how the rectification of the

records of older units' National Insurance Basic Pension receipts in our survey changes the Pension's vertical equity and redistributive impact. Given our assumptions, we find that random and systematic errors raise very sizeably the level of income support granted, and similarly augment the degree of exerted vertical equity and redistribution, at the cost of greater horizontal inequity. Claiming inconveniences and imperfect take-up smother the bearing of income support, decreasing somewhat the amount of horizontal inequity and lowering the degree of vertical equity and redistribution exerted by income support below that initially predicted by the original tax and benefit analysis.

Table 5.1

Benefits, Costs and Net Benefits

total
(average over the relevant sample of expected claimants)

	Benefits	Ex ante expected costs	Costs incurred	Net benefits
Total	22889 (35.49)	4643 (7.20)	3920 (6.08)	18969 (29.41)
Pensioners	8598 (29.96)	1817 (6.33)	1560 (5.44)	7038 (24.52)
Self- employed	179 (34.29)	72 (13.79)	64 (12.26)	115 (22.03)
Single parents	2442 (35.39)	248 (3.59)	201 (2.91)	2241 (32.48)

Table 5.2

Simulation of Changes in the Administration of Supplementary Benefits

total
(change)

	Expected costs incurred	Expected benefit	Expected number entitled	Expected number claiming	Take-up rate %	Expected net benefit
£1.02 ex ante increase in expected costs	4456 (536)	22773 (-116)	806.73 (0)	630.30 (-14.72)	78.1 (-1.8)	18317 (-652)
£1 ex ante decrease in entitlement	3787 (-133)	22104 (-785)	791.83 (14.90)	631.53 (-13.49)	79.8 (-0.1)	18317 (-652)

Table 5.3
Income Support and Equity

	Kakwani Index ¹	Average Benefit as Proportion of Final Net Income	Vertical Equity	Total Horizontal Inequity ²	Final Gini Coefficient
(a) Original NI Basic Pensions ³	1.033	0.0491	0.0508	0.018	0.289
(b) Revised NI Basic Pensions	1.014	0.0442	0.0448	0.018	0.289
(c) Calculated SB Entitlement	1.314	0.0268	0.0353	0.018	0.289
(d) Revised SB Entitlement	1.318	0.0305	0.0402	0.018	0.286
(e) Revised SB Entitlement with Random Administrative Errors	1.301	0.0349	0.0454	0.021	0.283
(f) Benefit Level, Imperfect Take-up	1.307	0.0316	0.0413	0.021	0.287
(g) Net Benefit Level, Imperfect Take-up	1.311	0.0255	0.0334	0.019	0.292

¹ This indicates the difference $G_X - I_{B,X}$.

² Indicates the difference $G_N - I_{N,X}$. For the two lines (a) and (b) of figures on pensions, the calculated SB entitlement is used to compute the index of total horizontal inequity exerted by all taxes and benefits. For all other lines, the revised NI basic pensions are applied (in conjunction with the varying SB variables).

³ For the two lines (a) and (b) of figures on pensions, the calculated SB entitlement is used.

Figure 5.1

Social Welfare and Costs to Claiming

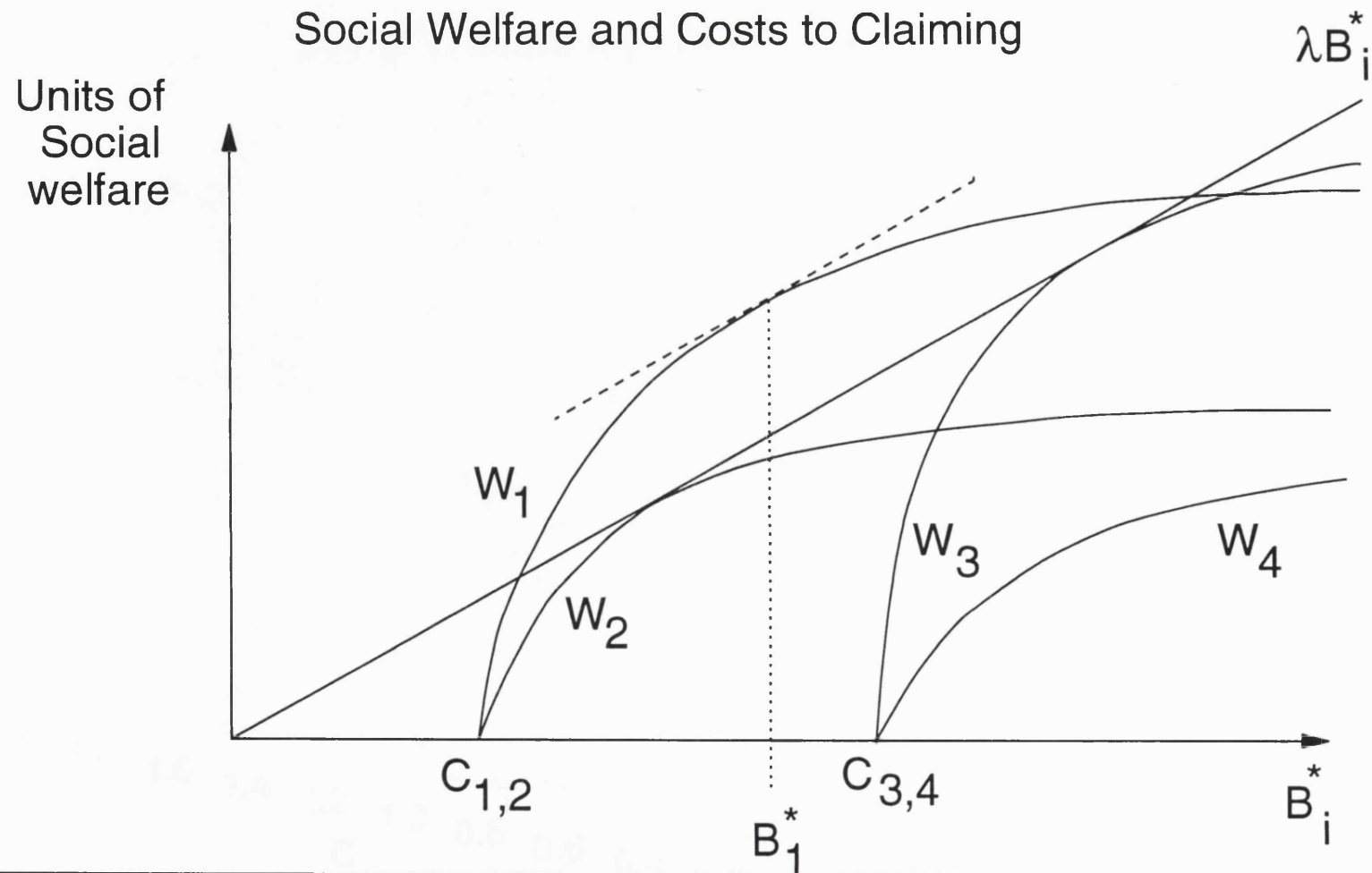


Figure 5.2

The Optimal Set of Claimants
and Levels of State Expenditure

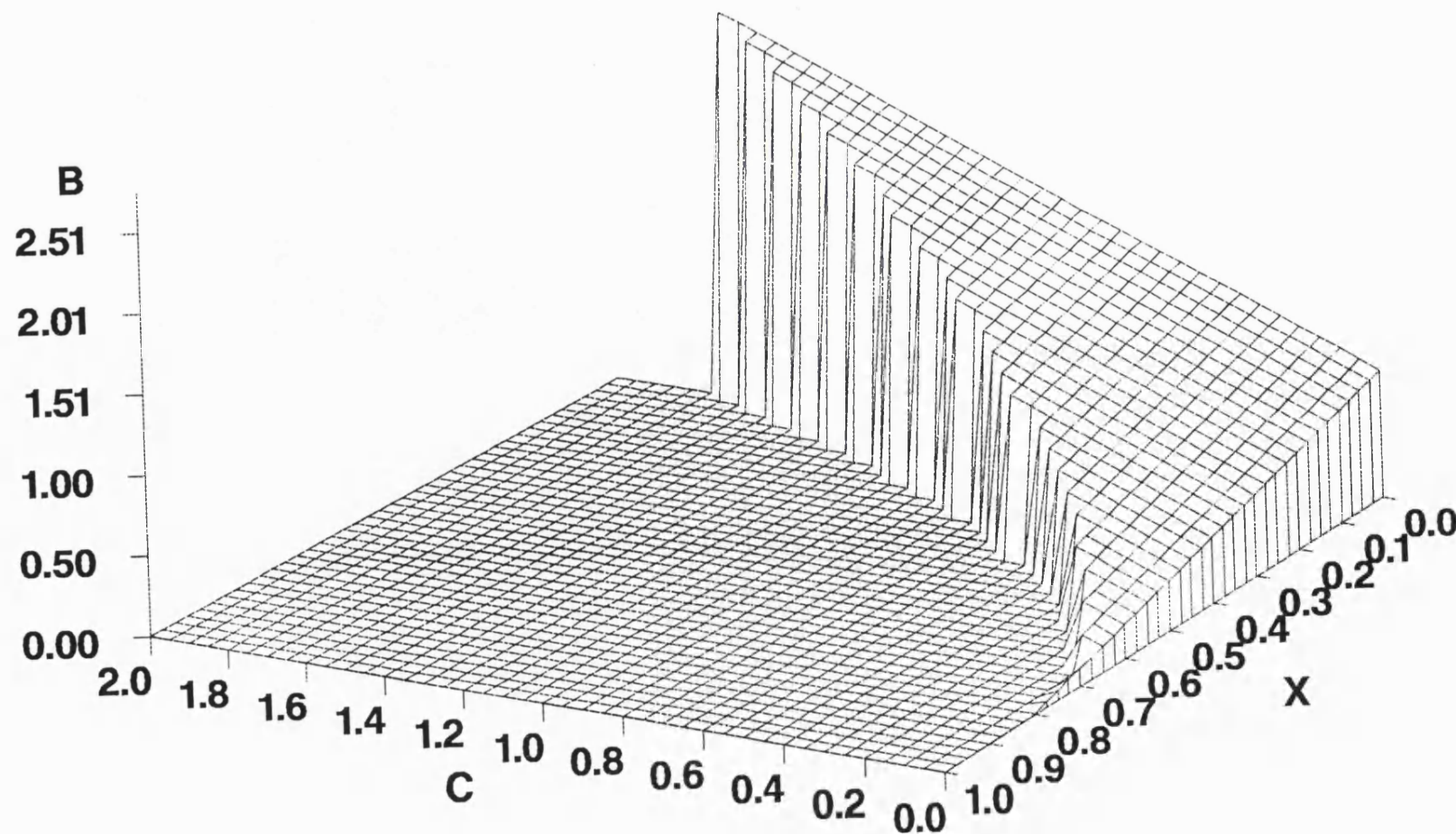


Figure 5.3
Income Support Efficiency

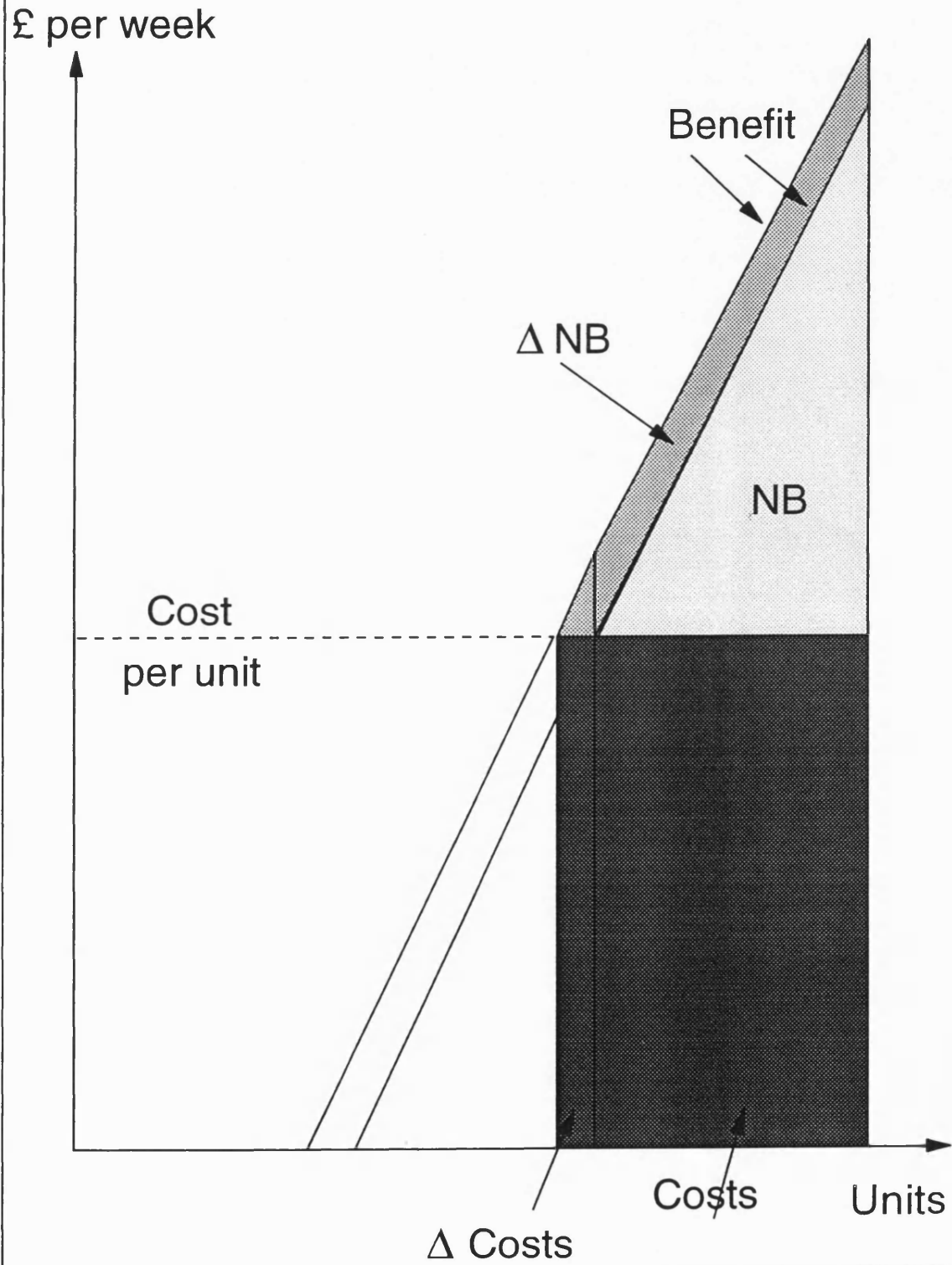
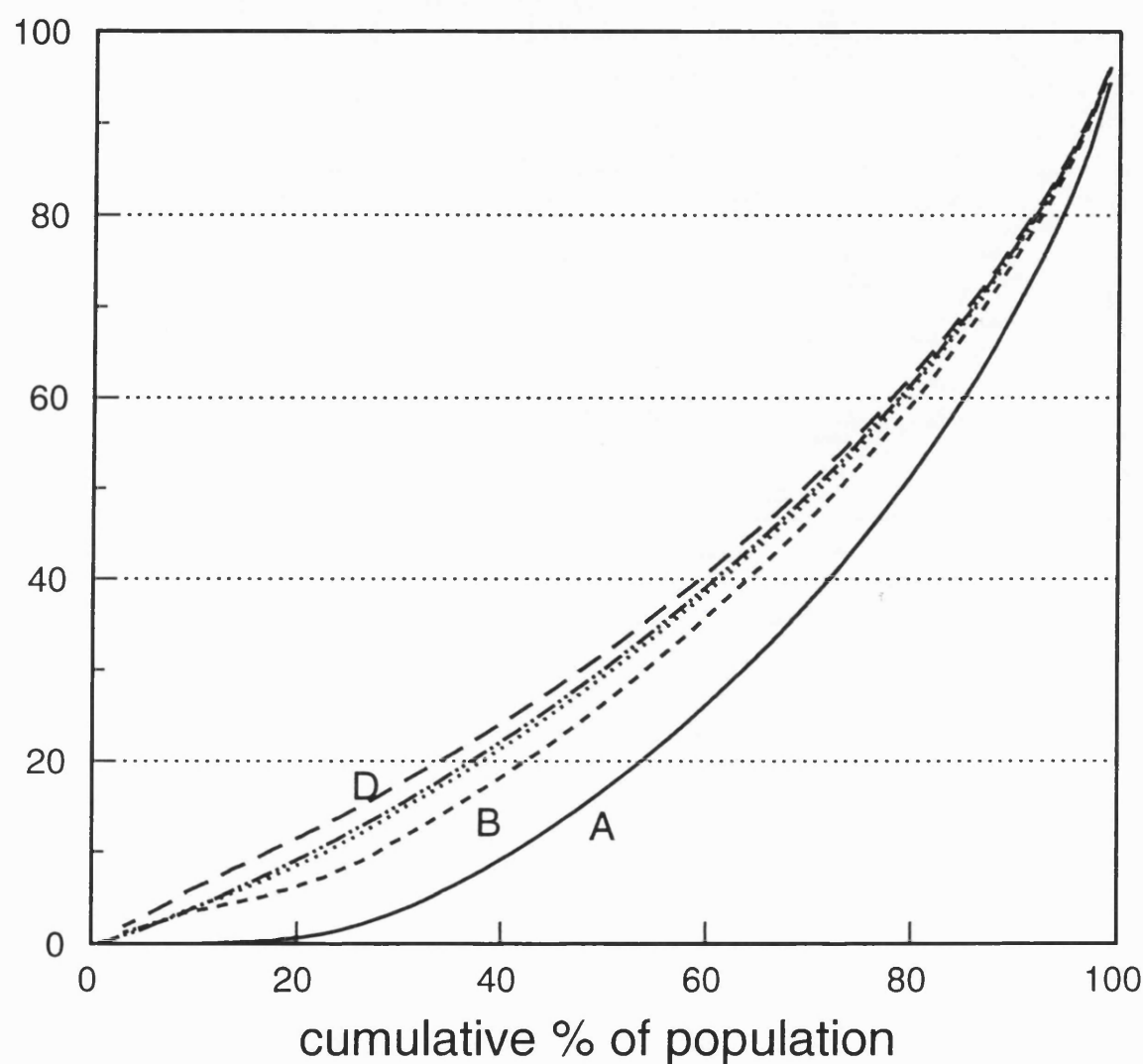


Figure 5.4

Income Redistribution

With Administrative Errors and Contracting Costs

cumulative % of total income



A: Original Income

B: A plus Various Benefits
and Taxes

C: B plus Adjusted
NI Pension

D: C plus Revised SB Entitlement
and Random Administrative Errors

E: C plus Net Benefit,
with Income Ranked in Order of E

The income distributions are ranked in the order
of the level of original income, unless
indicated

Conclusion

We are now at the finish of our enquiry into the impact of personal taxes and state and social security benefits in modern economies. We modestly believe to have fulfilled three major goals, but for each we ought to highlight honestly the major weaknesses of our analysis.

Firstly, we have provided a unified approach to the depiction of the progressivity, redistribution and equity of taxes and benefits. We have, as far as was reasonable, strived to identify the separate effects of individual taxes and benefits, focusing our empirical application on the system prevailing in Britain in 1985. There are, however, some important shortcomings in this applied work. Our definition of income is at best incomplete and our failure to embrace various imputed income streams (*e.g.*, those for home ownership) can lead to misleading conclusions. We only consider a temporal snapshot of a society's units and we do not ponder the effect of taxes and benefits over a life-cycle or across generations. Neither do we provide evidence on the statistical reliability of our results, especially for those results that lend themselves more easily to normative statements. We believe that our measure of original income is a good indicator of what income streams would prevail in the absence of taxes and benefits, but this ignores what are plausibly very important general equilibrium and behavioural forces. In particular, because agents' economic behaviour will normally adapt to policy adjustments in such a way as to improve on the first-round, fixed-behaviour, outcome, the assumption of behavioural unyieldingness will often underestimate the gains and overestimate the losses in welfare of the imposition of taxes and benefits. Finally, the role of the state is far from being limited to the

setting of personal taxes and income transfers; to present a more comprehensive picture of such a role, one would obviously wish to incorporate indirect taxes, expenditures in kind, the provision of public goods, the effect of government regulations, etc.. Interesting extensions of our work would also include the determination of optimal state support when "notches" are allowed. In so doing, one would likely account for both redistributive and efficiency concerns, and for the need to suscite as much as is reasonably possible a degree of autonomy and labour market integration on the part of state support recipients. One would also wish to derive measures of progressivity and vertical equity which would -- unlike those belonging to the class of measures of Chapter II -- not be biased by the presence of reranking and which would generate less ambiguous indicators of the redistributive impact of taxes and benefits.

Secondly, we have been relatively successful at constructing and implementing an econometric model that furthers our understanding of the take-up of state benefits in several important ways. A striking feature is our ability to generate estimates of entitlement discrepancies and inconveniences to claiming out of regular and broad survey data. The presence of such entitlement divergences raises important questions about both the reliability of using sample micro data for the analysis of taxes and benefits and the accuracy of the government's assessment of state support entitlement. The availability of micro data on **revised** administrative eligibility and non-eligibility (out of a second and more careful application process for some potential claimants, say) could obviously enrich our analysis and make possible the relative identification of the size of errors made by the analyst *versus* those made during the first enquiry by the government

agency. Uncertainty of entitlement on the part of potential state support claimants is likely to be a most important take-up determinant, and a good and feasible next step would be to model it explicitly and consistently. Including a labour supply behavioural component to one's status as a potential claimant would plausibly reveal lesser costs to seeking state support for those more likely to be entitled to it and thus for those working less or simply out of work. This is because eligibility, in addition to taking up or not, is also a choice variable, and low costs to claiming make it more attractive to increase entitlement by decreasing work efforts. Plausible errors and discrepancies on hours of work and levels of savings ought also to be modelled since they enter crucially into the determination of eligibility to Income Support in Britain. They also compound the importance of sound modelling in the analysis of state support.

Thirdly, we recognise explicitly the imperfections of the government's redistributive intervention. We do not discuss much the presence of administrative expenses, which can be easily documented, but we focus rather on those contracting costs which potential state support claimants must bear and on the allocative errors made in the process of granting income support. Our analysis potentially suffers here from flaws analogous to those described above, especially on the validity of our use of "original income" data and on the statistical reliability of some of our results. An important extension of our theoretical findings would be that of determining the optimal redistributive policy as a function of original income, administrative costs and the accuracy of the distributional information which the state is able to use in assessing the socio-economic characteristics of units.

We believe that the analytical and empirical framework developed in this doctoral thesis can also be usefully carried to other areas of applied economic analysis. In particular, our enquiry into levels of benefits and contracting costs lends itself nicely to a large array of economic applications. The closest conceptually to our own work is probably the desire of firms to qualify or not for business subsidies of various sorts, when such subsidies carry an application burden as well as economic distortions to the firms. Seeking state subsidies in such circumstances then amounts to considering the costs to be lower than the value of the state stipend. By a similar token, the availability of sufficiently suitable data would help price the inconveniences of tax evasion and elusion by private and corporate tax units and thus assist the government in designing optimal tax policies. The benefit to "cheating the government" can be straightforwardly (although imperfectly) appraised by the amount of taxes saved on income concealed or redirected to "offshore facilities", but there are also numerous transaction and opportunity costs (as well as the obvious fines if caught in illegal dealings) of hiding from the taxman.

Evident examples outside the world of public finance include consumption and investment behaviour in the presence of transaction, "shopping" and investment costs. The forward returns, besides being possibly uncertain (and imperfectly observable by the microanalyst), then have to be discounted by equally imperfectly observable costs. Similarly, the net benefit of collusion between firms depends on a level of benefit derived from such behaviour (which the analyst may believe to be non-positive) and on the extent of the presence of various contracting and collusive costs. All such examples have in common the

concept of an (at least partly) observable decision based on the value of an imperfectly observable net benefit, for which the benefit may be estimated or proxied with errors by an analyst. As illustrated by our work, however, such studies can carry important positive and normative conclusions.

Appendix A: Statistical Notes on the Use of the 1985 Family

Expenditure (FES) Survey Data

The FES is a continuous enquiry into the expenditure and income of private households in the United Kingdom (UK), carried out by the Office of Population Censuses and Surveys on behalf of the Department of Employment. The annual sample is about 11,000 households, representing roughly 1 in 2000 of all UK households, and of which around 70% (67% in 1985) accept to cooperate. People living in hostels, hotels, boarding houses and institutions are excluded. Evidence on differential response rates has been gathered over the years¹, suggesting that (among others) those without children, the self-employed, the older and the owner-occupiers had a lower response rate. In general, however, the FES provides an excellent and very reliable source of detailed knowledge about the characteristics -- including the labour supply behaviour and the distribution of the labour and non-labour revenues -- and the income of a large number of individuals. Full information on the FES (including copies of the questionnaire and the annual response analysis) can be obtained from the Department of Employment and from the Office of Population Censuses and Surveys, Social Survey Division, London.

By agreement with the Department of Employment, conditional access to the FES data for the purpose of my doctoral work was granted following an application to the ESRC Data Archive at the University of Essex. 1985 was the

¹ See, for instance, Kemsley, Redpath and Holmes (1980), Atkinson and Micklewright (1983), Redpath (1986), Atkinson, Micklewright and Stern (1988), and the empirical evidence presented in Chapter IV.

most appropriate year for which such data could be used at STICERD from the time at which this research was started. We concentrate exclusively on the use of the FES data gathered from April 8th to October 7th, 1985, these dates encompassing the policy period starting after the 1985 budget and ending with various autumn reviews to the tax and transfer systems.

The FES definition specifies that a household comprises one person living alone or a group of people living at the same address, having meals prepared together and with common housekeeping. We disaggregate households into income units (variable A008) as defined by the FES, and assume generally that these correspond to those units relevant to the appropriate computation of taxes and benefits, making explicit allowances when it is clear (such as in the case of Supplementary Benefits) that the consideration of different social categories matters. Because of this, heads of units will encompass all single person income units, including many of the young adults still living at home. People who are under 18 years of age and unmarried are classified as children. Dependent children are those below the age of 16 or those below 19 and in full-time education. We indicate below the way in which some of the variables of our tax and benefit model were derived using the 1985 FES data: "A..." denotes a FES variable appearing in the information codes of the household schedule, and a variable "F..." belongs to the income/expenditure codes of the same schedule.

We obtain the level of the last gross pay by the following formula: $F303 + F305 + F306 + F422 - F388 * (A234 = 1) + F422 + F424 - F429 + F316 + F320 + F312$. This excludes statutory sick pay but includes the value of coke and coal and of the meals and vouchers provided by the employer. The hourly wage is then found by

dividing this pay level by A216 [when different from zero, otherwise we estimate the hourly wage using the procedure described in Duclos (1992b)], the actual number of hours worked for the last pay. Original or gross income includes, among other variables, interests and dividends (F371, F376, F378, F374), labour income, non-BP pension receipts, statutory sick pay (F378), allowances from friends and former spouses (F352, F385), and private benefits and bus tokens (F366, F163). Self-employment income equals $F327 + F3276$ and incorporates the value of self-supplied goods, and self-employment status comes from the variable A203.

A010 gives the age at which full-time education ceased and, combined to age, can therefore also provide a rough indication of the amount of work experience gained by members of the sample. The current receipt of unemployment benefits is shown by A223.

Mortgage interest is taken as declared (F130 and F150). If the household does not know the amount of principal repaid along with the interest, it is assumed that it is null; mortgage interest is then $F201 + F202 + F203 + F204 + F205$. Similarly, when a unit does not know whether the interest paid was gross or net of MITR, we suppose that it was net of the subsidy. We take for the nominal interest rate on mortgages the value of 13.47%, a figure provided by the Association of Building Societies for the mean of the 1985 mortgage rates. Using this, we can impute the value of the unit's mortgage, of which the interest of only the first £30,000 qualify for mortgage relief.

To determine the level of savings for the estimation of eligibility to Supplementary Benefits (SB), we use the variable A284 -- which includes SAYE

and the national saving certificates and premium bonds held -- to which we also combine the information derived from the created variable BST. The level of capital imputed for the receipt of interests and dividends BST is obtained by grossing BST up by inverse of 0.06, the assumed average return to savings accounts and relevant shares. Long-term SB are granted to those units of which one of the members is aged 60 or over, or to those in receipt of benefits or pensions indicating the occurrence of special needs: A259, A238, F340. To find the size of the additional requirements, we use the condition that any of A238, F417, F421, F340, A225, A234, A259 be greater than zero or that age surpass 85. To define the level of the housing requirement not covered by Housing Benefits but rather part of SB, we use mortgage interest as discussed above as well as A163, F050, F232, F070 and F233.

Taxes retained under the composite rate scheme are computed from F376, F378, F371, F374 and using as an approximation to the composite rate the value of the basic tax rate, 0.30. Non-labour taxable income includes the amount of pensions exceeding NI basic pensions -- the computation being based on F342, F343, F344, F345, F346, F347 and F351 -- and on interests and dividends received, statutory sick pay, allowances from friends and absent spouses (F352, F385), widow's benefit (F339) and secondary earnings. Non-labour non-taxable income includes the value of redundancy payments (F356), war benefits (F340), various invalidity, sickness and injury allowances (F369, F325, F363, F418, F367, F417, F421), school meals and free milk (F259, F260, F263), income of dependent children, scholarships (F209, F210) and Training Opportunity Scheme (TOPS) and YOPS allowances.

Entitlement to National Insurance basic pensions (BP) -- i.e, not including the graduated or earnings-related pensions -- is equal to the variable F338 in most cases, unless F338 exceeds the 1985 BP values, in which case entitlement is reduced to the flat-rate part of the NI scheme. We therefore can allow somewhat for the deflating impact on the level of BP entitlement of an incomplete NI contributory record. Other necessary adjustments to BP are described in Chapter IV -- where it is necessary to calculate a "floor" level of BP received, taking into account the possibility that a receipt of SB may have been mistaken for one of BP. Whenever possible, we then use the declaration of the receipt of some spouse's BP addition as an indicator of some minimum BP indeed received by the unit. Otherwise, the minimum BP receipt is set to zero. Finally, "various (observed) benefits" include the value of those benefits whose allocation is recorded in the FES but which are modelled separately in our tax and benefit model; they comprise war benefits, widow's benefit, invalidity pensions and allowances, individual injury disablement pension, National Insurance sickness and industrial injury benefit, mobility and attendance allowances, TOPS and YOPS grants, school meals and milk, war benefits and other minor additions to non-labour income.

The complete set of SAS and FORTRAN files used to generate the 1985 tax and benefit model is available upon request.

Appendix B: Note on the Major Changes to the British Tax and Benefit System Between 1985 and 1992

The methodological and theoretical contributions of the thesis are applied to the taxes and benefits and to the socio-economic circumstances of the United Kingdom in 1985. As such, the empirical findings presented in this work are intrinsically linked to the particular conditions of the UK at that time, and we would expect the applied results to be different were they to be derived from other distributions of income and household characteristics. Our empirical findings also hinge strongly on the structure of the 1985 British tax and benefit system, although the system's major features show surprising recurrence across time and societies.

The British tax and benefit system underwent substantial changes in the years 1985 to 1992, the year in which this thesis was completed. The number of personal income tax brackets decreased from six in 1985, with marginal tax rates ranging from 30% to 60%, to only two by 1988 to 1991, with a basic tax rate equal to 25% and a higher one of 40% affecting no more than around 5% of the British families. In the 1992 budget, an additional tax bracket of 20% was introduced in order to give greater credibility to the governing party's official long term policy of lowering the basic rate of tax from 25% to 20%. Independent taxation of wife and husband was introduced in 1990, with a Married Couple's Allowance being available in addition to single allowances to offset part of the husband's and the wife's income.

The "composite rate of tax" on interest and dividends has since 1985 been abolished, with the implication that those not liable to pay income taxes may

receive interest and building society dividend payments at the gross rates. Tax incentives for savings channelled in "TESSA's" were introduced, and the tax generosity of Personal Equity Plans (PEPS) was enhanced. Mortgage relief at the higher rate was stopped (with the effect that the subsidy is now payable at the same rate for all tax and non-tax payers) and the £30,000 limit was not raised in line with the general and housing price levels. The rate structure on National Insurance Contributions varied for contributions on low levels of earnings – the changes removing slowly over time the non-convexity of taxing all earnings once a lower earnings limit has been reached – but the upper earnings limit has remained until now, being increased only in line with the price level. Domestic rates were displaced by the Community Charge (better known as the "poll tax"), of which a minimum of 20% had to be paid by all adults (including those for whom all domestic rates had been previously reimbursed by the state) and which will itself be replaced next year by a "new" hybrid, the Council Tax.

The nature and the generosity of Social Security benefits also evolved. The value of child benefits was kept nominally constant over almost the whole 1985-1992 period and was thus eroded in real terms. The "earnings rule" on the availability of National Insurance pensions was removed, making their payment not conditional upon the current earnings status of pensioners. Contribution rules for eligibility to unemployment benefits became stiffer. The regulations governing the operation of State Earnings Related Pension (SERP) changed drastically, although the full impact of such a reform will not be felt until some more years yet.

The Supplementary Benefit (SB) scheme was replaced in 1988 by a new

Income Support programme. The entitlement and payment rules were somewhat simplified, and "long-term needs" were supplanted by premia for belonging to various socio-economic categories. The benefit became much less generous (and often unavailable) for people under the age of 25. A less generous "Social Fund" replaced the single need and urgent needs payments previously available under SB. Standards common to the administration of Family Credit (which then replaced the less broad Family Income Supplement) and Housing Benefits (for which steeper withdrawal rates were imposed) were established regarding capital and income limits. The administration and the allocation of the various state benefits thus became more homogenous and the prevalence of implicit rates of taxes and withdrawals above 100% could thus be more easily averted.

The changes in the rate structure of personal income taxation and the provision of greater tax and shares incentives imply some strong shifts towards a generally less progressive and redistributive tax system, although a net assessment would also take into account the impact of the changes in mortgage subsidisation and of the new 20% tax bracket. Notwithstanding the reforms described above, the design of state benefits remains, however, broadly the same. Benefits have unevenly kept pace with the general increase in prices, and have almost uniformly fallen behind the rise in earnings and average living standards. Stronger considerations of cost "efficiency" and economic incentives have furthermore led to a tightening of eligibility rules and to a more direct targeting of state support towards those considered to be in greatest need. In the light of the empirical results of this thesis and of those of other works, these recent features can almost certainly be predicted to decrease the direct redistributive

impact of the state.

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